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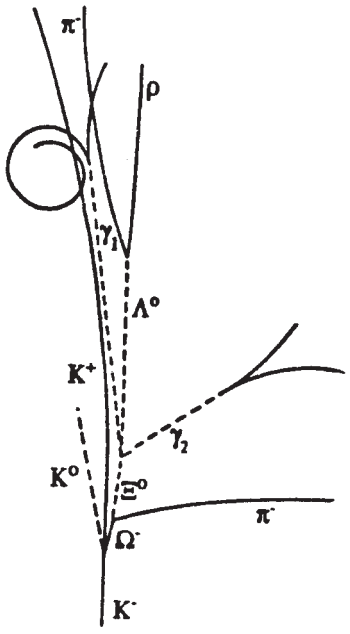

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VOLUME 44, NUMBER 2, JUNE 2021

QUANTUM AND CLASSICAL UNIVERSE AND ANTI-UNIVERSE, 109
H. Strusny
Hohenschoenhauser Str. 1
10369 Berlin, Germany
ADDITIONAL CONSERVED QUANTITIES OF WEBER'S FORCE LAW?, 169
Alan Aversa
ANOMALOUS MAGNETIC MOMENTUM OF MUONS IN FNAL ON THE BASIS OF QUANTIZE SPACE AND TIME THEORY, 181
Hamid Reza Karimi
Electrical Engineering Department
Islamic Azad University, South Tehran Branch
Tehran, 17776-13651, Iran
HOW AND WHY THE DECAY MODES OF BEAUTY QUARKS INTO LEPTONS DECAY IN LHCb ON THE BASIS OF THE QUANTIZED SPACE AND TIME THEORY, 193
Hamid Reza Karimi
Electrical Engineering Department
Islamic Azad University, South Tehran Branch
Tehran, 17776-13651, Iran
EFFECTIVE POTENTIAL OF MUONIC AND TAUONIC ATOMS OF HELIUM AND HELIUM LIKE ATOMS, 201
Saddam Husain Dhobi
Department of Physics, Patan Multiple Campus
Tribhuvan University, Lalitpur-44700, Nepal
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Innovative Ghar Nepal of Nepal, Lalitpur-44700, Nepal

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QUANTUM AND CLASSICAL UNIVERSE AND ANTI-UNIVERSE

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Abstract

The beginning of the quantum universe and anti-universe, which are based on the gravitation, results from two equivalent energy uncertainties via one quantum fluctuation of the vacuum (origin), so that they expand in the opposite time directions. For the estimation of the parameters of their beginning, we use the smallest possible massive particle rest energies of photons and gravitons. We show that the end of the quantum universe (anti-universe) as well as the beginning of the classical universe (anti-universe) are simultaneously determined by the positive (negative) Planck time in the future (past), respectively. Therefore, the classical universe and the classical anti-universe, which are based on the inflation model and the Friedmann-Lemaitre Equations, must expand again in the opposite time directions, so that we have found a reasonable solution of the problem of the separation of the matter and the antimatter, using the general description of the radiation-dominated and matter-dominated universe (anti-universe). We explain the reason for the massive rest energy of photons and gravitons via the quantum field theory. We show that at the Planck length the cosmic inflation event of the strong-electroweak symmetry breaking is plausible. All these results are supported by cosmic energy considerations via a well-established prediction of the quantum field theory. In the quantum cosmology, the quantization of the space-time leads to its separation. We justify the new thermal equilibrium via the massive particle formation in the quantum gravity. We compare the particle energy densities of the quantum and classical universe (anti-universe), whereat we estimate the rest energy of charged Higgs bosons. We solve precisely and uniquely the known monopole problem.

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1 Introduction

In Refs. [1-3], we have introduced the total {"massless" ($0 \to R_{BB} \le R \le R_{Pl}$) and massive ($R > R_{Pl}$)} universe, where $R_{BB} << R_{Pl}$ and $R \le R_{Pl}$ describe the space distance of the big bang (BB) and of the "massless" universe, whereas R_{Pl} and $R > R_{Pl}$ are the Planck distance and the known scale factor of the massive universe, respectively. The "massless" universe is determined by the gravitation [1-3], whereas the massive universe is defined by a new inflation model [1-5] and the Friedmann-Lemaitre Equations [1-13]. In Refs. [2-5], we have shown that the "massless" and the massive universe has a Euclidian geometry. In our most recent work [1], we show in detail that all results, given for this universe [1-3], are also valid for the (time-reversal) anti-universe.

Besides, in the work [1], we have given a detailed review for the transition from the final state of the universes in the direction to the big bang as a start of an eternal cyclic evolution, so that these two states are bound with each other. The reason of this relationship are zero-point oscillations, which arise from the complete decay of the sterile neutrinos [1-5, 14] between the final state of these universes and the big bang. The cause of the final state of the universe is a mean negative acceleration [1-5] interpreted as Pioneer anomaly [1-5, 15, 16]. Additionally, we have confirmed the parameters of the big bang and the lifetime of the sterile neutrinos [1], so that we can corroborate the eternal cyclic evolution of the universes [1], whereat we have derived a solution, in which the universe and the anti-universe begin their existence via two equivalent energy uncertainties by one quantum fluctuation of the vacuum [1], so that they must expand in the opposite time directions. Thus, the anti-universe has its existence in the past, whereas the universe exists in the future [1]. We describe the dark matter and dark energy by the special properties of the sterile neutrinos [1]. The existence of a new thermal equilibrium is supported by the light neutrinos [1]. Further, in Ref. [1], we have estimated the rest energy of the photons and the gravitons in accordance with their limiting values [17]. The existence of the rest energy of the photons, which must be identical with the rest energy of the gravitons, is

confirmed by the measured general galactic magnetic field [10, 18] and the theoretical magnetic neutrino moment [1-5, 11, 19].

Theoretical data [1], derived via the rest energy of the photons, permit far-reaching conclusions about the various Hubble "constants" for the different epochs of evolution of the universe. Therefore, the Hubble expansion rate $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, which in Ref. [8] was assumed as the present Hubble "constant" of the universe, is interpreted in the work [1] as the present "Hubble constant" of the cosmic microwave background (CMB), since it was derived by the Planck observations [13] from the measurement of the cosmic back ground radiation, which was formed at $3.72 \cdot 10^5$ years after the big bang [8]. Thus, this new CMB Hubble "constant" $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ can be used further as basis for all hitherto existing considerations for the evolution of the universe, i.e. it must also determine all Hubble "constants" of the universe. Consequently, in the $\Lambda\mathrm{CDM}$ model, we assumed that this present CMB value $H_0 = 67.3\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ must also yield a new Hubble constant for the beginning of the accelerated (acc) expansion of the universe, so that we have obtained this new Hubble "constants" to $H_{\mathrm{acc}} = 95.0\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [1]. Then, after the accelerated expansion, we have a slow linear (lin) expansion [1-3] with the Hubble "constant" $H_{\mathrm{lin}} = 0.843\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ [1], derived by aid of the astronomical unit changing [2-6, 20]. Consequently, the accelerated expansion must be decelerated, so that we must observe the present Hubble "constant" $H_{\mathrm{acc},0}$ by the condition $H_{\mathrm{acc}} > H_{\mathrm{acc},0} > H_0 >> H_{\mathrm{lin}}$. The calculated value $H_{\mathrm{acc},0} = 74.0\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is excellently confirmed by the most recent observations of Riess [21] in 2019, so that we need no new physical assumptions.

Via time-dependent vacuum energy densities or cosmological "constants" (see, e.g., Refs. [1, 22]), we have solved generally the discrepancy between the vacuum energy of the Planck scale and the present dark energy as a continuous transition in accordance with the quantum field theory [1]. The same solution was obtained from the big bang to the final state of the universe, using again the Planck scale [1]. We have also demonstrated this

behaviour from the big bang to the Planck scale as well as from Planck scale to the final state of the massive universes via the X, Y gauge bosons [1-6], the Higgs boson and the electron [1].

It is usual to explain the properties of the universe by the quantum cosmology (quantum gravity) as well as the "classical" formalism of the relativistic Friedmann-Lemaitre Equations and of the inflation. Therefore, in this work, the "massless" universe (anti-universe) is explained as the quantum universe (anti-universe), whereas the massive universe (anti-universe) is treated as the classical universe (anti-universe), respectively.

However, unfortunately, to this day, this quantum gravity is still highly incomplete and yields therefore no reliable predictions because the connection between the cosmological "constant" and the vacuum energy density is not clear [7]. Therefore, in the works [2, 3], by aid of the virtual matter of the quantum vacuum, we have derived a simple solution for the long-sought four-dimensional quantum gravity.

In the present review, we analyze in detail the most important results of the works [1-6].

Thus, this work is organized as follows. In Sec. 2, we define the necessary quantities and constants for the universe and the anti-universe. In Sec. 3, we consider generally the quantum universe and anti-universe. In Sec. 4, via the Friedmann-Lemaitre Equations and the new inflation model (Sec. 4.1), we describe, for example, the classical radiation-dominated and matter-dominated universe as well as anti-universe (Sec. 4.2). In Sec. 5, we explain the existence of the massive rest energy of the photons and the gravitons. In Sec. 6, we take into account the pair number of the sterile neutrinos at the zero-point oscillations. In Sec. 7, we treat specially the cosmic inflation. In Sec. 8, we perform several cosmic energy considerations via a well-established prediction of the quantum field theory. In Sec. 9, we justify the new thermal equilibrium via the massive particle formation in the quantum cosmology, considering the quantization of the space-time. In Sec. 10, we compare the particle energy densities of the quantum and classical universe (anti-universe), whereat we estimate the rest energy of charged Higgs bosons. In Sec. 11, we solve exactly the monopole problem. In Sec. 12, we give a short summary. The physical constants, used in this work, are given in Refs. [8, 11].

2 Quantities and constants for universe and anti-universe

The Table I gives the most important known properties of particles and antiparticles. The Table II yields the most important quantities and constants for the universe and the anti-universe, derived via the known properties of the particles and the antiparticles (in connection with the time reversal of the time component of the 4-vectors and the tensors).

TABLE I. The most important known properties of particles and antiparticles.

Property	Particle	Anti-particle
energy	E	E
mass	M	M
time	t	$-t$
momentum	p	$-p$
velocity	v	$-v$
elementary electric charge	q	$-q$

TABLE II. The most important quantities and constants for the universe and the anti-universe, derived via the known properties of the particles and the antiparticles (in connection with the time reversal of the time component of the 4-vectors and the tensors).

Quantity	Universe	Anti-universe
energy	E	E
temperature	T	T
Boltzmann constant	k	k

Table II, continued.

Quantity	Universe	Anti-universe
mass density	ρ	ρ
gravitational constant	G_N	G_N
distance, scale factor	R	R
acceleration	$\ddot{R}$	$\ddot{R}$
velocity including c	$\dot{R}$	$-\dot{R}$
reduced Planck constant	$\hbar$	$-\hbar$
time	t	$-t$
age	t_0	$-t_0$ [a)]
Hubble expansion rate	H	$-H$
present CMB Hubble constant	H_0	$-H_0$
scale factor of H_0	h	$-h$
ionized charges	q	$-q$

[a)] For the positive time direction, we obtain t_0 instead of $-t_0$.

In work [3], for the anti-universe, all assumptions derived from the known properties of the antiparticle, are correct. Unfortunately, the work [2] gives here the negative distance $-R=-|\pm R|$ and scale factor $-R$ {as a result of a careless application of the CPT invariance (acting only in the three-dimensional space)} instead of the correct positive three-dimensional gravitational space distance $R=|\pm R|$ and four-dimensional scale factor R. Thus, we have corrected already these two quantities in Ref. [1].

TABLE III. The most important, critically analyzed, present, cosmological parameter values [8] of the classical universe. In this work, H_0 is interpreted as the present CMB Hubble "constant" (see also Sec. 1). [a] Calculated by us for $h = 0.673 \pm 0.012$. [b] Different fits [8]

Quantity	Symbol, equation	Value
present day CMB temperature	T_0	$2.7255(6)\ \mathrm{K}$
present day CMB Hubble expansion rate	H_0	$67.3 \pm 1.2\ \mathrm{km\,s^{-1}\,Mpc^{-1}} =$ $= h \times (9.777752\ \mathrm{Gyr})^{-1} =$ $= (2.181 \pm 0.039) \cdot 10^{-18}\ \mathrm{s^{-1}}$
scale factor for Hubble expansion rate	h	0.673 ± 0.012
critical present density of the universe	$\rho_{0C} = 3H_0^2/8\pi G_N$	$1.05375(13) \cdot 10^{-5} h^2\,(\mathrm{GeV}/c^2)\ \mathrm{cm^{-3}} =$ $= 4.77(17) \cdot 10^3\,(\mathrm{eV}/c^2)\ \mathrm{cm^{-3}}$
baryon (proton) density of the universe	$\Omega_b = \rho_b/\rho_{0C}$	$0.02207(27)\,h^{-2} = 0.0487(23)$ [a], $0.0499(22)$ [b]
(cold) dark matter density of the universe	$\Omega_{dm} = \rho_{dm}/\rho_{0C}$	$0.1198(26)\,h^{-2} = 0.265(15)$ [a], $0.265(11)$ [b]
dark energy density of the universe	$\Omega_\Lambda = \rho_\Lambda/\rho_{0C}$	$0.685^{+0.017}_{-0.016}$

TABLE III, continued.

Quantity	Symbol, equation	Value
pressureless matter density of the universe	$\Omega_{\mathrm{m}} = \Omega_{\mathrm{b}} + \Omega_{\mathrm{dm}}$	$0.315^{+0.016}_{-0.017}$
CMB radiation density of the universe	$\Omega_\gamma = \rho_\gamma / \rho_{0\mathrm{C}}$	$2.473 \cdot 10^{-5} (T/2.7255\ \mathrm{K})^4 h^{-2} =$ $= 5.46(19) \cdot 10^{-5}$
curvature	$\Omega_{\mathrm{tot}} = \Omega_{\mathrm{m}} + \Omega_\Lambda + \cdots$	$0.96^{+0.4}_{-0.5}$ (95% CL); $1.000(7)$ (95% CL; CMB + BAO)
sum of neutrino masses	$\sum m_\nu c^2$	< 0.23 eV
neutrino density of the universe	$\Omega_\nu = \rho_\nu / \rho_{0\mathrm{C}}$	$< 0.0025\, h^{-2}$; < 0.0055
redshift of matter-radiation equality	z_{eq}	3360 ± 70
redshift of photon decoupling	z_{dec}	1090.2 ± 0.7
redshift at half reionization	z_{reion}	11.1 ± 1.1
age of the universe	t_0	13.81 ± 0.05 Gyr = $= (4.358 \pm 0.016) \cdot 10^{17}$ s
Hubble length	$R_0 = c/H_0$	$0.9250629 \cdot 10^{26}\, h^{-1}$ m = $= (1.375 \pm 0.025) \cdot 10^{26}$ m [a]

In Refs. [1-5], we have proved that these cosmological parameters of Tables III are excellently described by particle-defined present cosmological parameter values, which were calculated by the light neutrino density parameters as well as the ratios of the relativistic energy to the rest energy of the SUSY GUT particles. We have summarized these particle-defined present cosmological parameters in Table IV. In Ref. [1-3], by aid of the data of Tables I and II, we have shown that these particle-defined present cosmological parameters are also valid for the anti-universe. Because of the excellent agreement between these observed (Table III) and calculated (Table IV) cosmological parameters, we apply always the cosmological parameter values of Table III in the present work at the calculations. In this work, the results of the most recent analysis of the observed present cosmological parameters and the 3 neutrino oscillation parameters of Ref [17] are not applied, since within the error limits these most recent data agree well with the corresponding cosmological parameters of Ref. [8] and the 3 neutrino oscillation parameters of Ref [11]. Therefore, a renewed calculation of all astrophysical parameters, determined in Refs. [1-6], where they were derived by aid of the cosmological parameters of Ref. [8] and the 3 neutrino oscillation parameters of Ref. [11], is first necessary, if new better measurements are available, since within the error limits the application of the cosmological parameters and 3 neutrino oscillation parameters from Ref. [17] gives similar results. Thus, in this work, we use always as basis the cosmological parameters from Table I, given by Ref. [8].

Table IV. The estimated particle-defined present cosmological parameter values for the classical universe and anti-universe according to Refs. [1-5] (compare with Tables III). [a)] Universe: "+"sign, anti-universe: "−"sign.

Symbol, equation	Value for universe and anti-universe
$\lvert\pm h\rvert$ [a)]	$0.6736^{+0.0105}_{-0.0096}$
$\lvert\pm H_0\rvert$ [a)]	$100\,h\ \mathrm{km\ s^{-1}Mpc^{-1}} = 67.36^{+1.05}_{-0.96}\ \mathrm{km\ s^{-1}Mpc^{-1}} =$ $= h\times(9.777752\,\mathrm{Gyr})^{-1} = (2.183^{+0.034}_{-0.031})\cdot 10^{-18}\ \mathrm{s^{-1}}$

Table IV, continued.

Symbol, equation	Value for universe and anti-universe
$\lvert \pm t_0 \rvert$ [a), b)]	$13.82^{+0.06}_{-0.07}$ Gyr
$R_0 = c/H_0$	$(1.373^{+0.020}_{-0.021}) \cdot 10^{26}$ m
$\rho_{0C} = 3H_0^2/8\pi G_N$	$(4.78^{+0.15}_{-0.14}) \cdot 10^3 (\mathrm{eV}/c^2)\,\mathrm{cm}^{-3}$
Ω_{tot}	1
$\Omega_{\mathrm{b}} = \rho_{\mathrm{b}}/\rho_{0C}$	$0.02211^{+0.00089}_{-0.00091}\, h^{-2} = 0.0487 \pm 0.0020$
$\Omega_{\mathrm{dm}} = \rho_{\mathrm{dm}}/\rho_{0C}$	$0.1202^{+0.0059}_{-0.0051}\, h^{-2} = 0.265^{+0.013}_{-0.012}$
$\Omega_{\Lambda} = \rho_{\Lambda}/\rho_{0C}$	$0.311^{+0.014}_{-0.013}\, h^{-2} = 0.686^{+0.020}_{-0.021}$
$\Omega_{\mathrm{m}} = \rho_{\mathrm{m}}/\rho_{0C}$	$0.1423^{+0.0059}_{-0.0052}\, h^{-2} = 0.314^{+0.013}_{-0.012}$
$\Omega_{\gamma} = \rho_{\gamma}/\rho_{0C}$	$(2.4728 \pm 0.0025) \cdot 10^{-5} h^{-2} = (5.45^{+0.15}_{-0.17}) \cdot 10^{-5}$
$\Omega_{\nu} = \rho_{\nu}/\rho_{0C}$	$(6.35^{+0.16}_{-0.14}) \cdot 10^{-4} h^{-2} = (1.402^{+0.040}_{-0.044}) \cdot 10^{-3}$
$\sum m_{\nu} c^2$	$(5.97^{+0.14}_{-0.13}) \cdot 10^{-2}$ eV
$z_{\mathrm{eq}} = \dfrac{2\Omega_{\mathrm{m}}}{N(T)\Omega_{\gamma}} - 1$	3423^{+107}_{-94}
z_{dec}	1090.7
$z_{\mathrm{reion}}(\nu_{\tau})$	$11.60^{+0.31}_{-0.26}$

[b)] For positive time direction we obtain t_0 instead of $-t_0$.

3 The quantum universe and anti-universe

By the quantum gravity [1-3], for the quantum universe and anti-universe, via the gravitation law, using the quantum fluctuations of the quantum vacuum, we obtain the following expressions for the connection between the distance R as well as the photon energy kT or the particle (P) rest energy $E_0(\mathrm{P})$:

$$R = \frac{G_N}{c^4} kT = \frac{\hbar c}{E_{\mathrm{Pl}}^2} kT = \frac{G_N}{c^4} E_0(\mathrm{P}) = \frac{\hbar c}{E_{\mathrm{Pl}}^2} E_0(\mathrm{P}), \tag{3.1}$$

for the distance:

$$R = ct, \tag{3.2}$$

for the vacuum energy density:

$$\rho_{\mathrm{vac}}(R)c^2 = \widehat{\rho}_{\mathrm{vac}}\, c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^2}{\hbar c R^2}, \tag{3.3}$$

for the cosmological "constant":

$$\Lambda = \widehat{\Lambda} = \frac{16}{45}\pi^3 \Omega_\gamma \frac{1}{R^2}, \tag{3.4}$$

for the particle number density:

$$n_{\mathrm{P}}(R) = \frac{\rho_{\mathrm{vac}}(R)c^2}{kT} = \frac{\rho_{\mathrm{vac}}(R)c^2}{E_0(\mathrm{P})} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{1}{R^3}, \tag{3.5}$$

for the kinetic photon energy:

$$E_{\mathrm{K}}(\gamma) = 2.701178\, kT = 2.701178 \frac{E_{\mathrm{Pl}}^2}{\hbar c} R \tag{3.6}$$

and for the thermal photon number density:

$$n_\gamma(R) = 2.701178 \frac{\rho_{\mathrm{vac}}(R)c^2}{E_{\mathrm{K}}(\gamma)} = \frac{\rho_{\mathrm{vac}}(R)c^2}{kT} = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{1}{R^3}, \tag{3.7}$$

where the quantities G_N, c, k, T, $\hbar$, E_{Pl} and Ω_γ describe the gravitational constant, the speed of light in vacuum, the Boltzmann constant, the absolute temperature, the reduced Planck constant, the Planck energy and the

radiation density of the universe (see Table III), respectively. Consequently, we have $n_{\mathrm{P}}(R) = n_{\gamma}(R)$, i.e. the particle number density and the thermal photon number density are equivalent.

Now, if we consider the photon rest energy $E_0(\gamma) \cong 1.563 \cdot 10^{-35}$ eV [1, 2] and the graviton rest energy $E_0(\mathrm{G}) \cong 1.563 \cdot 10^{-35}$ eV [1, 2] as the smallest possible particle energies in the form of $kT = E_0(\gamma)$ and $kT = E_0(\mathrm{G})$ as well as $E_0(\mathrm{P}) = E_0(\gamma)$ and $E_0(\mathrm{P}) = E_0(\mathrm{G})$ for the big bang (BB) of the quantum universe, Eq. (3.1) yields

$$R = R_{\mathrm{BB}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2} E_0(\gamma) = \frac{G_N}{c^4} E_0(\mathrm{G}) = 2.069 \cdot 10^{-96} \text{ cm}, \tag{3.8}$$

Then, because of Eq. (3.2), the big bang of the quantum universe takes place at the time

$$t = t_{\mathrm{BB}} = \frac{R_{\mathrm{BB}}}{c} = 6.901 \cdot 10^{-107} \text{ s}. \tag{3.9}$$

Using the results of the Table II, we can assume in accordance with Ref. [1] that the beginning of the anti-universe (antimatter) and the universe (matter) is a result of two equivalent energy uncertainties by one quantum fluctuation of the vacuum according to the uncertainty relation in form of

$$\widetilde{E}_{\mathrm{BB}} = -\hbar/2(-t_{\mathrm{BB}}) = 4.769 \cdot 10^{90} \text{ eV} \tag{3.10}$$

and

$$\widetilde{E}_{\mathrm{BB}} = \hbar/2t_{\mathrm{BB}} = 4.769 \cdot 10^{90} \text{ eV}, \tag{3.11}$$

respectively. At $t \to 0$ (origin), these two energy uncertainties disappear by annihilation. The total energy $E_{\mathrm{BB}} = \hbar/t_{\mathrm{BB}} = 9.537 \cdot 10^{90}$ eV must be provided by the vacuum $E \to 0$ (origin). An absolute value $E = 0$ is unrealizable.

Taking the results (3.3) and (3.4) as well as (3.8), for the big bang, we find the (time-dependent) vacuum energy density and the (time-dependent) cosmological constant to

$$\rho_{\mathrm{vac}}(R_{\mathrm{BB}})c^2 = \frac{2}{3}\Omega_{\gamma} \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^2}{\hbar c R_{\mathrm{BB}}^2} \cong 4.227 \cdot 10^{247} \text{ eV cm}^{-3} \tag{3.12}$$

and

$$\Lambda = \Lambda_{\mathrm{BB}} = \frac{16}{45}\pi^3 \Omega_\gamma \frac{1}{R_{\mathrm{BB}}^2} \cong 1.406 \cdot 10^{192}\ \mathrm{m}^{-2}\,, \tag{3.13}$$

respectively. Using the results (3.5 and (3.7) as well as (3.8), for the big bang, we obtain the particle (graviton) number density and the thermal photon number density to

$$n_{\mathrm{P}}(R_{\mathrm{BB}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\mathrm{BB}}^3} \cong 2.704 \cdot 10^{282}\ \mathrm{cm}^{-3} \tag{3.14}$$

and

$$n_\gamma(R_{\mathrm{BB}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\mathrm{BB}}^3} \cong 2.704 \cdot 10^{282}\ \mathrm{cm}^{-3}\,, \tag{3.15}$$

respectively. The expressions (3.14) and(3.15) agree excellently with the results (3.20) and (3.21) in Ref. [1], where the initial values of $n_{\mathrm{P}}(R_{\mathrm{BB}})$ and $n_\gamma(R_{\mathrm{BB}})$ were estimated to $2.706 \cdot 10^{282}\ \mathrm{cm}^{-3}$ via the complete conversion of the dark energy [1] into the decay products of the sterile neutrinos in the final state of the universe [1], so that they have also permitted the determination of R_{BB} and t_{BB} [1] in accordance with (3.8) and (3.9). Consequently, using the results of Ref. [1], we can say that the end determines the beginning.

Analogously, if we consider correspondingly the Planck energy as the greatest possible particle energies in the form of $kT = E_{\mathrm{Pl}}$ and $E_0(\mathrm{P}) = E_{\mathrm{Pl}}$ for the end of the quantum universe, Eq. (3.1) gives

$$R = R_{\mathrm{Pl}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2} E_{\mathrm{Pl}} = \frac{G_N}{c^4} E_{\mathrm{Pl}} = 1.6162 \cdot 10^{-33}\ \mathrm{cm}\,. \tag{3.16}$$

Then, because of Eq. (3.2), the end of the quantum universe takes place at the time

$$t = t_{\mathrm{Pl}} = \frac{R_{\mathrm{Pl}}}{c} = 5.39106 \cdot 10^{-44}\ \mathrm{s}\,. \tag{3.17}$$

Using the results of the Table II in Eqs. (3.16) and (3.17), we can assume in accordance with Ref. [1] that the end of the quantum anti-universe is defined by the positive distance $R = R_{\mathrm{Pl}} = 1.6162 \cdot 10^{-33}$ cm and the negative speed of light $-c$ of the vacuum, i.e. it must be characterized by the negative time $t = t_{\mathrm{Pl}} = R_{\mathrm{Pl}}/(-c) = -5.39106 \cdot 10^{-44}$ s.

Therefore, more generally, because of Ref. [1], the end of the quantum universe (anti-universe) must simultaneously determine the beginning of the classical universe (anti-universe).

Consequently, the quantum anti-universe has its existence at the positive distances in the past for the negative time direction $-t_{\mathrm{Pl}} \leftarrow -t \leftarrow -t_{\mathrm{BB}} \leftarrow 0$, whereas the quantum universe exists at the positive distances in the future for the positive time-direction $0 \rightarrow t_{\mathrm{BB}} \rightarrow t \rightarrow t_{\mathrm{Pl}}$. Thus, the classical universe must exist for $t \geq t_{\mathrm{Pl}}$, whereas the classical anti-universe is defined by $t \leq -t_{\mathrm{Pl}}$. On this way, we have found the long-sought solution for the important problem of the separation of the antimatter (anti-universe) and the matter (universe). To the classical universe (anti-universe), we will return in Sec. 4.

Then, taking the results (3.3) and (3.4) as well as (3.16), in accordance with Ref. [1], for the end of the quantum universe and the quantum anti-universe, we find the (time-dependent) vacuum energy density and the (time-dependent) cosmological constant to

$$\rho_{\mathrm{vac}}(R_{\mathrm{Pl}})c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\mathrm{Pl}}^2}{\hbar c\, R_{\mathrm{Pl}}^2} \cong 9.63 \cdot 10^{121}\ \mathrm{eV\,cm^{-3}} \tag{3.18}$$

and

$$\Lambda = \Lambda_{\mathrm{Pl}} = \frac{16}{45}\pi^3 \Omega_\gamma \frac{1}{R_{\mathrm{Pl}}^2} \cong 2.30 \cdot 10^{66}\ \mathrm{m^{-2}}\,, \tag{3.19}$$

respectively. Using the results (3.5 and (3.7) as well as (3.16), at the Planck distance, we obtain correspondingly the particle number density and the thermal photon number density to

$$n_{\mathrm{P}}(R_{\mathrm{Pl}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\mathrm{Pl}}^3} \cong 5.67 \cdot 10^{93}\ \mathrm{cm^{-3}} \tag{3.20}$$

and

$$n_\gamma(R_{\mathrm{Pl}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\mathrm{Pl}}^3} \cong 5.67 \cdot 10^{93}\ \mathrm{cm^{-3}}\,, \tag{3.21}$$

respectively.

For example, if we assume the proton rest energy $E_0(\mathrm{p}) = 9.38272 \cdot 10^8\ \mathrm{eV}$, the assumptions $kT = E_0(\mathrm{p})$ and $E_0(\mathrm{P}) = E_0(\mathrm{p})$ yield correspondingly the

following results, which must lie reasonably between the values (3.8) to (3.21), i.e. we have

$$R = R_{\mathrm{p}} = \frac{\hbar c}{E_{\mathrm{Pl}}^2} E_0(\mathrm{p}) = 1.242 \cdot 10^{-52}\ \mathrm{cm}, \tag{3.22}$$

$$t = t_{\mathrm{p}} = \frac{R_{\mathrm{p}}}{c} = 4.143 \cdot 10^{-63}\ \mathrm{s}, \tag{3.23}$$

$$\rho_{\mathrm{vac}}(R_{\mathrm{p}})c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\mathrm{Pl}}^2}{\hbar c\, R_{\mathrm{p}}^2} = 1.173 \cdot 10^{160}\ \mathrm{eV\,cm^{-3}}, \tag{3.24}$$

$$\Lambda = \Lambda_{\mathrm{p}} = \frac{16}{45}\pi^3 \Omega_\gamma \frac{1}{R_{\mathrm{p}}^2} = 3.902 \cdot 10^{104}\ \mathrm{m^{-2}}, \tag{3.25}$$

$$n_{\mathrm{P}}(R_{\mathrm{p}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\mathrm{p}}^3} = 1.250 \cdot 10^{151}\ \mathrm{cm^{-3}} \tag{3.26}$$

and

$$n_\gamma(R_{\mathrm{p}}) = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{1}{R_{\mathrm{p}}^3} = 1.250 \cdot 10^{151}\ \mathrm{cm^{-3}}. \tag{3.27}$$

4 The classical universe and anti-universe

4.1 The Friedmann-Lemaitre Equations and the new inflation model

In Refs. [2-5], we have generally derived $\mathrm{k} = 0$ by $\Omega_{\mathrm{tot}} = 1$ and $\Omega_{\mathrm{tot}}(z) = 1$, so that according to Ref. [1] in the ΛCDM model the Friedmann-Lemaitre Equations are given by

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N}{3}\rho + \frac{c^2}{3}\Lambda \tag{4.1}$$

and

$$\frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3}\left(\rho + \frac{3P}{c^2}\right) + \frac{c^2}{3}\Lambda , \tag{4.2}$$

where the quantities $H = H(t)$, $R = R(t)$, ρ, Λ and P describe the Hubble parameter, the scale factor, the mean mass density, the cosmological constant and the isotropic pressure, respectively.

It is usual to introduce the mass density [1-4, 6-9, 11]:

$$\rho = \rho_\Lambda + \rho_{\rm m} + \rho_{\rm r} = \left[\Omega_\Lambda + \Omega_{\rm m}(R_0/R)^3 + \Omega_{\rm r}(R_0/R)^4\right]\rho_{0C} =$$

$$= \left[\Omega_\Lambda + \Omega_{\rm m}(1+z)^3 + \Omega_{\rm r}(1+z)^4\right]\frac{3H_0^2}{8\pi G_N}, \tag{4.3}$$

where according to Table III the quantity ρ_{0C} is the present, critical density

$$\rho_{0C} = \frac{3H_0^2}{8\pi G_N}. \tag{4.4}$$

The radiation density parameter $\Omega_{\rm r}$ is defined by

$$\Omega_{\rm r} = \frac{1}{2}N(T)\,\Omega_\gamma , \tag{4.5}$$

where the statistical particle weights $N(T)$ are given in Refs. [2-4, 11]. For example, in Eq. (4.5), at temperatures $T << 0.5\,\mathrm{MeV}$, $N(T)$ has the value

$$N(T) = 3.362644 , \tag{4.6}$$

since the only relativistic species are photons at the temperature T as well as massless or nearly massless neutrinos and antineutrinos of three different types at the temperature $T_\nu = (4/11)^{1/3}T$ because of the entropy conservation [2-4, 7, 9]. Thus, according to Refs. [7, 9], $N(T)$ of Eq. (4.5) has the value (4.6) at the present CMB temperature $T = T_0 = 2.7255\,\mathrm{K}$ (see Table III) and increases at higher temperatures (see Refs. [2-4, 11]).

Consequently, the present classical universe contains a relic background of light neutrinos [7] with the temperature

$$T_{\nu,0} = \left(\frac{4}{11}\right)^{1/3}T_0 = 1.9454\,\mathrm{K}. \tag{4.7}$$

In Eq. (4.5), the radiation density parameter Ω_γ [1-4, 6-9] is defined by

$$\Omega_\gamma = \frac{\rho_0(\gamma)}{\rho_{0C}} = 5.46 \cdot 10^{-5} \tag{4.8}$$

with the radiation density [1-4, 6-9, 11, 12]:

$$\rho_0(\gamma) = \{\pi^2/15(\hbar^3 c^5)\}(kT_0)^4 . \tag{4.9}$$

In an expanding universe, the observed wavelength λ of the light emitted (e) from a distant source with λ_e, is shifted towards the red. Via the Friedmann-Robertson-Walker metric [1-4, 6-9, 11, 12], this so-called redshift z is defined for these photons by

$$1 + z = \frac{\lambda}{\lambda_e} = \frac{R_0}{R} = \frac{T}{T_0}, \tag{4.10}$$

where R_0 and R are the scale factors of absorption and emission of light, respectively. Then, by Eqs. (4.3) and (4.4), the relativistic formula (4.1) yields

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = H_0^2\left[\Omega_\Lambda + \Omega_m\left(\frac{R_0}{R}\right)^3 + \Omega_r\left(\frac{R_0}{R}\right)^4\right] \tag{4.11}$$

with the general solution

$$t = t(z) = \frac{1}{H_0}\int_0^{1/(1+z)} \frac{dx}{x\,(\Omega_\Lambda + \Omega_m x^{-3} + \Omega_r x^{-4})^{1/2}}, \tag{4.12}$$

where $x \equiv (R/R_0) = (1/[1+z])$.

The corresponding parameter values, which were partly above introduced, are summarized in the Table III, which contains the critically analyzed observed present cosmological parameter values for the classical universe according to Ref. [8].

In Refs. [1-3], we have shown that for the description of the classical anti-universe the present cosmological parameter values of Table III are also valid, except the values H_0, h, t_0 and t, which are here negative. This behavior can be attributed to the properties of the parameters of the Table IV [1-3] or to the assumption of the second sign "$-$" of the root (4.11)

It is usual to take apart Eq. (4.3) in the dominant density terms in dependence on the redshift evolution (4.10).

Then, for example, if the universe expands adiabatically, the entropy per comoving volume is constant, so that we can describe the mass density of the radiation-dominated universe for $z \geq 10^5$ or $T \geq 3 \cdot 10^5$ K by

$$\rho = \rho_r = \tfrac{1}{2} N(T)\, \Omega_\gamma (1+z)^4 \rho_{0C} = \tfrac{1}{2} N(T)(1+z)^4 \rho_0(\gamma). \qquad (4.13)$$

By the new inflation model [1-6], for the redshift evolution $1+z$, we have

$$\widetilde{R} = \frac{R}{1+z_M} = \frac{\widetilde{R}_0}{1+z} \qquad (4.14)$$

$$R = (1+z_M)\widetilde{R} = \frac{R_0}{1+z}, \qquad (4.15)$$

where $\widetilde{R}$ and R characterize the scale factors for the emission of light of prior to and after the inflation, whereas $\widetilde{R}_0$ and R_0 define the constant scale factors for the absorption of light before and after the inflation, respectively.

According to Refs. [1-6], via the present critical mass density (4.4), for the conditions (4.14) and (4.15), the quantities $\widetilde{R}_0$ and R_0 were found to

$$\widetilde{R}_0 = (R_{Pl}\, c/H_0)^{1/2} = (4.713 \pm 0.042) \cdot 10^{-5} \text{ m} \qquad (4.16)$$

and

$$R_0 = c/H_0 = (1.375 \pm 0.025) \cdot 10^{26} \text{ m}. \qquad (4.17)$$

By the results (4.14) and (4.15), which define immediately the inflation, the redshift condition

$$1 + z_M = \frac{T_M}{T_0} = \frac{\widetilde{R}_0}{R_{Pl}} = \frac{R_0}{\widetilde{R}_0} = (2.916 \pm 0.026) \cdot 10^{30} \qquad (4.18)$$

enlarges the early classical universe ($R_{Pl} \leq \widetilde{R} \leq \widetilde{R}_0$) by the enormous factor $1 + z_M = 2.916 \cdot 10^{30}$ to the late classical universe ($\widetilde{R}_0 \leq R$), whereat T_M is the photon temperature, which is defined by the magnetic monopoles (M) via their rest energy [1-6]

$$E_0(M) = kT_M = (1+z_M)\, kT_0 = (6.849 \pm 0.063) \cdot 10^{17} \text{ GeV}. \qquad (4.19)$$

4.2 The classical universe (anti-universe)

According to Refs. (1-3], the general vacuum energy density or the cosmological "constant" of the classical universe were found to

$$\rho_{\rm vac}(T)\,c^2 = \frac{1}{3}\frac{\pi^2}{15}\frac{(kT)^4}{(\hbar c)^3}\frac{1}{N(T)} \tag{4.20}$$

or

$$\Lambda = \widetilde{\Lambda} = \frac{8\pi G_N}{c^2}\rho_{\rm vac}(T) = \frac{8}{45}\pi^3\frac{(kT)^4}{E_{\rm Pl}^2(\hbar c)^2}\frac{1}{N(T)}, \tag{4.21}$$

respectively. For $R = R_{\rm Pl}$ or $T = T_{\rm Pl}$, according to Refs. [1-6], we have

$$N(T) = \frac{1}{2\Omega_\gamma}, \tag{4.22}$$

so that the early classical universe is also defined by the corresponding values

$$\rho_{\rm vac}(R_{\rm Pl})\,c^2 = \frac{2}{3}\Omega_\gamma\frac{\pi^2}{15}\frac{E_{\rm Pl}^2}{\hbar c\,R_{\rm Pl}^2} \cong 9.63\cdot 10^{121}\ {\rm eV\,cm^{-3}} \tag{4.23}$$

or

$$\Lambda = \Lambda_{\rm Pl} = \frac{16}{45}\pi^3\Omega_\gamma\frac{1}{R_{\rm Pl}^2} \cong 2.30\cdot 10^{66}\ {\rm m^{-2}} \tag{4.24}$$

in accordance with (3.18) and (3.19). Because of Table II, we have additionally $R_{\rm Pl} = (-c)(-t_{\rm Pl})$, so that the results (4.23) and (4.24) are also valid for the anti-universe.

For example, the classical universe and anti-universe are determined by the Friedmann equation (4.1) via the densities (4.3) and (4.4), so that because of the Tables II and IV they are defined by the expression (4.12), which yields the time directions $t \to +t$ and $t \to -t$ for the classical universe and anti-universe, respectively. Consequently, the velocities $\dot{R} = dR/d(-t)$ of the classical anti-universe must possess a negative sign, so that we get here the negative velocities in contrast to the positive velocities $\dot{R} = dR/dt$ of the classical universe.

Then, using, for example, the general solutions [1-4, 6] of Eqs. (4.1) and (4.13) via the results (4.14) to (4.18) for the early ($R_{\rm Pl} \le \widetilde{R} \le \widetilde{R}_0$ or $\widetilde{R}_0 \ge \widetilde{R} \ge R_{\rm Pl}$) and the late ($R \ge \widetilde{R}_0$ or $\widetilde{R}_0 \le R$) radiation-dominated classical universe and anti-universe ($z \ge 10^5$), we have always the positive scale factors

$$\widetilde{R} = (2\,N(T)\,\Omega_\gamma)^{1/4}(R_{\rm Pl}\,c\,t)^{1/2} = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\,(t_{\rm Pl}\,t)^{1/2} \tag{4.25}$$

and

$$R = (2\,N(T)\,\Omega_\gamma)^{1/4}(R_0\,c\,t)^{1/2} = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{t}{H_0}\right)^{1/2}, \tag{4.26}$$

respectively. This fact is clear because we must consider $-\dot{R}$ and the sign "$-$" of the root of Eq. (4.1), which is equivalent to $-H_0$ (see Table II).

Then, the corresponding velocities ($z \ge 10^5$) are given by

$$\dot{\widetilde{R}} = \frac{d\widetilde{R}}{dt} = \pm\frac{1}{2}(2\,N(T)\,\Omega_\gamma)^{1/4}(R_{\rm Pl}\,c/t)^{1/2} =$$

$$= \pm\frac{1}{2}c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{t_{\rm Pl}}{t}\right)^{1/2} \tag{4.27}$$

and

$$\dot{R} = \frac{dR}{dt} = \pm\frac{1}{2}(2\,N(T)\,\Omega_\gamma)^{1/4}(R_0\,c/t)^{1/2} =$$

$$= \pm\frac{1}{2}c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{1}{H_0\,t}\right)^{1/2}, \tag{4.28}$$

where according to Table II the sign "+" and the sign "$-$" are valid for the corresponding universe and anti-universe, respectively.

Taking $(\dot{R}/R)^2 = 1/(2t)^2$ in Eq. (4.1) together with Eqs. (4.8) and (4.9), we can here derive the known expression ($z \ge 10^5$)

$$t = t(z) = \pm \frac{1}{2}\left(\frac{90}{8\pi^3}\right)^{1/2} \hbar E_{\mathrm{Pl}} \frac{1}{\sqrt{N(T)}\,(kT)^2} =$$

$$= \pm \frac{(2.42035 \pm 0.00015)}{\sqrt{N(T)}}\left(\frac{\mathrm{MeV}}{kT}\right)^2 \mathrm{s} =$$

$$= \pm \frac{(3.25936 \pm 0.00021)}{\sqrt{N(T)}}\left(\frac{10^{10}\,\mathrm{K}}{T}\right)^2 \mathrm{s} =$$

$$= \pm \frac{(0.438773 \pm 0.000028)}{\sqrt{N(T)}}\left(\frac{10^{10}}{1+z}\right)^2 \mathrm{s}, \qquad (4.29)$$

where according to Table II the sign "+" and the sign "−" are again valid for the corresponding universe and anti-universe, respectively.

Using the results $N(T) = 1/2\,\Omega_\gamma$ (see Eq. (4.22)) and $kT = E_0(\mathrm{M})$ {see Eq. (4.19)} in Eq. (4.29), for the beginning of the early classical radiation-dominated universe and anti-universe, we obtain $t = \pm t_{\mathrm{Pl}}$, so that Eqs. (4.25) and (4.27) yield correspondingly $\tilde{R} = R_{\mathrm{Pl}}$ and $\dot{\tilde{R}} = \pm \frac{1}{2} c$.

For $z_{\mathrm{eq}} >> z >> 1$, the matter-dominated universe and anti-universe are described by Eqs. (4.1) as well as (4.3) and (4.4) in the form of

$$\left(\frac{\dot{R}}{R}\right)^2 \approx \Omega_{\mathrm{m}} H_0^2 \left(\frac{R_0}{R}\right)^3. \qquad (4.30)$$

Similarly, using again the data of Tables II and IV, Eq. (4.30) has the solutions

$$R \approx (3/2)^{2/3}\,\Omega_{\mathrm{m}}^{1/3} R_0 (H_0 t)^{2/3}, \qquad (4.31)$$

$$\dot{R} = \frac{dR}{dt} \approx \pm \frac{2}{3}(3/2)^{2/3}\,\Omega_{\mathrm{m}}^{1/3} \frac{c}{(H_0 t)^{1/3}} \qquad (4.32)$$

and

$$t = t(z) \approx \pm \frac{2}{3 H_0 \left[\Omega_{\mathrm{m}} (1+z)^3\right]^{1/2}}, \qquad (4.33)$$

where according to Table II the sign "+" and the sign "–" are valid for the matter-dominated universe and anti-universe, respectively.

The continuation of these problems leads to the final state of the classical universe and anti-universe, which however is treated in detail in work [1], so that we complete here these considerations.

Consequently, for the classical universe, we have the positive time direction (from the positive Planck time to the future), which is valid for all fundamental physical processes in the universe, so that we have here an expanding universe with scale factors greater than zero as well as positive velocities $c > v > 0$. Correspondingly, for the classical anti-universe, we get a negative time direction (from the negative Planck time to the past as a result of the time reversal), which is valid for all fundamental physical processes in the anti-universe, i.e. we have here an expanding anti-universe with scale factors greater than zero as well as negative velocities $-c < -v < 0$ and the transitions $H_0 \to -H_0$, $t_{\mathrm{Pl}} \to -t_{\mathrm{Pl}}$ and $t \to -t$.

Thus, we have simply explained the long-sought problem of the separation between matter (classical universe) and antimatter (classical anti-universe).

The above-described solution of the existence of the universe and the anti-universe explains also the fact that all fundamental physical laws are time-invariant, since they must be valid not only in the universe but also in the anti-universe, i.e. all fundamental physical processes of the universe and the anti-universe take place in the positive and the negative time direction, respectively.

However, if we assume as usual alone the existence of the observed positive time direction for the anti-universe and the universe, this contracting anti-universe is existent only in the past and must disappear on its way in the future at the transition from $t = -t_{\mathrm{BB}}$ to $t = t_{\mathrm{BB}}$ as the beginning of an expanding universe, whereat the energy $\widetilde{E}_{\mathrm{BB}} = -\hbar/2(-t_{\mathrm{BB}})$ of the anti-universe must be applied to the universe according to $\widetilde{E}_{\mathrm{BB}} = \hbar/2t_{\mathrm{BB}}$ for its way in the future. In spite of the observed positive time direction, a such assumption is unrealistic because of the time invariance of all fundamental physical laws.

It is now clear that the results of the universe, derived in this work and the papers [1-6], are completely transferable to the anti-universe, using the data of Tables II and IV.

5 Derivation of the massive rest energy of photons and gravitons

Using Eq. (3.1) via the energy $E = kT$ in the form of

$$R = \frac{\hbar c}{E_{\mathrm{Pl}}^2} E, \tag{5.1}$$

which was derived by the gravitational potential energy in Refs. [2, 3], we get physically as greatest possible relativistic particle energy $E = E_{\mathrm{Pl}} = kT_{\mathrm{Pl}}$ at $R = R_{\mathrm{Pl}}$. Mathematically, in Eq. (5.1), we can also introduce $E = kT = 0$ and $R = 0$. However, for example, because of the quantum field theory, the absolute quantities $E = kT = 0$ and $R = 0$ are inaccessible values, i.e. the rest energy $E = 0$ for Photons and gravitons is forbidden.

Now, we prove the massive rest energy $E_0(\gamma) \cong 1.563 \cdot 10^{-35}$ eV for photons [1, 2] and the massive rest energy $E_0(\mathrm{G}) \cong 1.563 \cdot 10^{-35}$ eV [1, 2] for gravitons via the quantum field theory. In Ref. [1], we have found the limiting values of its time-dependent cosmological "constants" for the dark energy to

$$\Lambda = \Lambda_\Lambda = \frac{3\Omega_\Lambda}{R_0^2} = \frac{3\Omega_\Lambda}{c^2/H_0^2} =$$

$$= \frac{3\Omega_\Lambda}{c^2 \tau_{\mathrm{H}}^2} = 1.087 \cdot 10^{-52}\ \mathrm{m}^{-2}, \tag{5.2}$$

for the Planck units to

$$\Lambda = \Lambda_{\mathrm{Pl}} = \frac{16\pi^3}{45} \Omega_\gamma \frac{1}{R_{\mathrm{Pl}}^2} =$$

$$= \frac{16\pi^3}{45} \Omega_\gamma \frac{1}{c^2 t_{\mathrm{Pl}}^2} = 2.30 \cdot 10^{66}\ \mathrm{m}^{-2} \quad \text{(see also Eq. (3.19))} \tag{5.3}$$

and for the big bang to

$$\Lambda = \Lambda_{BB} = \frac{16\pi^3}{45}\Omega_\gamma \frac{1}{R_{BB}^2} =$$

$$= \frac{16\pi^3}{45}\Omega_\gamma \frac{1}{c^2 t_{BB}^2} = 1.406 \cdot 10^{192}\ \mathrm{m}^{-2} \quad \text{(see also Eq. (3.13))}. \tag{5.4}$$

Because of the new inflation model [1-3], we have a discontinuity in the evolution of the universe, so that the ratios

$$\frac{\Lambda_{PI}}{\Lambda_\Lambda} = \frac{16\pi^3}{135}\frac{\Omega_\gamma}{\Omega_\Lambda}\frac{\tau_H^2}{t_{PI}^2} \cong 2.119 \cdot 10^{118} \tag{5.5}$$

as well as

$$\frac{\Lambda_{BB}}{\Lambda_{PI}} = \frac{t_{PI}^2}{t_{BB}^2} \cong 6.10 \cdot 10^{125} \tag{5.6}$$

do not agree with the result of the quantum field theory, which predicts a value of about 10^{122} for the result (5.5) at a continuous evolution of the classical universe [12]. However, because of the inflation, we must simulate a continuous evolution for the quantum and classical universe by

$$(\Lambda_{BB}\Lambda_\Lambda)^{1/2}/\Lambda_\Lambda = (\Lambda_{BB}/\Lambda_\Lambda)^{1/2} =$$

$$= \left(\frac{16\pi^3}{135}\frac{\Omega_\gamma}{\Omega_\Lambda}\right)^{1/2}\frac{1}{H_0 t_{BB}} \cong 1.137 \cdot 10^{122}, \tag{5.7}$$

so that we obtain an excellent agreement with the prediction of the quantum field theory. In other words, the expression $(\Lambda_{BB}/\Lambda_\Lambda)^{1/2} = 1.137 \cdot 10^{122}$, which simulates the continuous evolution of the universe, corresponds to the geometric mean of the results (5.5) and (5.6).

Then, using the prediction of the quantum field theory, via Eq. (5.7), we find the quantized time to

$$t_{BB} = \left(\frac{16\pi^3}{135}\frac{\Omega_\gamma}{\Omega_\Lambda}\right)^{1/2}\frac{1}{H_0 \times 1.137 \cdot 10^{122}} \cong 6.90 \cdot 10^{-107}\ \mathrm{s} \tag{5.8}$$

in accordance with the result (3.9). Consequently, because of the result (5.8) Eq. (3.2) yields the quantized space distance

$$R_{\mathrm{BB}}=\left(\frac{16\pi^3}{135}\frac{\Omega_\gamma}{\Omega_\Lambda}\right)^{1/2}\frac{c}{H_0\times 1.137\cdot 10^{122}}\cong 2.069\cdot 10^{-96}\ \mathrm{cm}, \tag{5.9}$$

so that for photons and gravitons the expression (5.1) gives their massive rest energy via

$$E_0(\gamma)=E_0(\mathrm{G})=\frac{E_{\mathrm{Pl}}^2}{\hbar c}R_{\mathrm{BB}}=1.563\cdot 10^{-35}\ \mathrm{eV} \tag{5.10}$$

in accordance with the inverse Eq. (3.8). With that, via the prediction of the quantum field theory, we have proved the correctness of the massive rest energy of photons and gravitons, derived in Ref. [1].

6 The particle-antiparticle pair number at the zero-point oscillations

In this chapter, we demonstrate that the existence of the quantum universe ($0\le R_{\mathrm{BB}}\le R\le R_{\mathrm{Pl}}$) can be attributed to the zero-point oscillations [1], which according to Ref. [1] are also valid the classical universe.

Thus, at the big bang of the quantum universe, these zero-point oscillations must be excited via the greatest possible relativistic energy

$$E_{\mathrm{BB}}=2\widetilde{E}_{\mathrm{BB}}=\hbar c/R_{\mathrm{BB}}=\hbar/t_{\mathrm{BB}}=9.537\cdot 10^{90}\,\mathrm{eV} \tag{6.1}$$

because of the beginning of the quantum universe at $R_{\mathrm{BB}}=2.069\cdot 10^{-96}$ cm (see Eqs. (3.8) as well as (3.10) and (3.11)).

According to Ref. [1], we use de Broglie waves. In the extremely relativistic case, between the momentum p of the particle as well as its relativistic energy E, we have the connection

$$p=\frac{E}{c}. \tag{6.2}$$

Then, the de Broglie wavelength λ of the sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$ [1-5], which describe the joint origin of the dark matter and dark energy (see, e.g., Ref. [1]), is defined by

$$\lambda = 2\pi\frac{\hbar}{p} = \frac{\hbar c}{E/2\pi}. \tag{6.3}$$

For Eq. (6.3), we can write

$$\frac{\hbar c}{\lambda} = \frac{E}{2\pi}. \tag{6.4}$$

Consequently, we must consider $N_{\hat{\nu}\uparrow} = 3$ excited sterile neutrinos $\hat{\nu}_\Lambda$, $\hat{\nu}_{dm}$ and $\hat{\nu}_b$ according to Dirac's fermion-antifermion theory, so that we can introduce $E(\hat{\nu}_\Lambda) + E(\hat{\nu}_{dm}) + E(\hat{\nu}_b) \to E$ for $E(\hat{\nu}_\Lambda) = E(\hat{\nu}_{dm}) = E(\hat{\nu}_b)$ in Eqs. (6.3) and (6.4). Therefore, analogous to Eq. (6.4) we get the condition $\hbar c/\lambda(\hat{\nu}_\Lambda) = \hbar c/\lambda(\hat{\nu}_{dm}) = \hbar c/\lambda(\hat{\nu}_b)$ with $\lambda(\hat{\nu}_\Lambda) = \lambda(\hat{\nu}_{dm}) = \lambda(\hat{\nu}_b) \to \lambda$ for the quantum universe. The arrow $\uparrow$ describes their spin direction.

Analogously, these zero-point oscillations must excite simultaneously $N_{\hat{\bar{\nu}}\uparrow} = 3$ sterile anti-neutrinos $\hat{\bar{\nu}}$ in the anti-universe, so that we obtain $N = 3$ sterile neutrino-antineutrino pairs for these quantum universes. If we consider correspondingly still $N_{\hat{\nu}\downarrow} = 3$ sterile neutrinos and $N_{\hat{\bar{\nu}}\downarrow} = 3$ antineutrinos with opposite spin direction (Pauli exclusions principle), we have together $N = 6$ excited sterile neutrino-antineutrino pairs.

Considering the simultaneous application of Dirac's fermion-antifermion theory for the universe and the anti-universe, we must introduce $N = 12$ sterile neutrino-antineutrino pairs.

For the final state of the classical universe (anti-universe), we have assumed in Ref. [1] that the excitation energy (6.1) of the zero-point oscillations has an energy loss in form of the factor $(\Omega_{dm} + \Omega_\Lambda)^{1/6} = 0.9915$ by the effecting of the complete decay of 3 fundamental massive sterile neutrinos [1, 2] of the dark matter Ω_{dm} (see Table III) and the dark energy Ω_Λ (see Table III) into two massless decay products [1].

Therefore, for the N excited sterile neutrino-antineutrino pairs, we must introduce the real excitation (exc) energy $E_{exc}(N) = (\Omega_{dm} + \Omega_\Lambda)^{1/6} E_{BB}$, where $E_{BB} = 9.537 \cdot 10^{90}$ eV is defined by Eq. (6.1).

Consequently, at N excited sterile neutrino-antineutrino pairs, by the transition $E \to E_{\text{exc}}(N)$, Eq. (6.4) must be transformed correspondingly to

$$\text{a)} \quad (\Omega_{\text{dm}} + \Omega_{\Lambda})^{1/6} N \frac{\hbar c}{\lambda} = \frac{E_{\text{exc}}(N)}{2\pi},$$

$$\text{b)} \quad \lambda = \frac{(\Omega_{\text{dm}} + \Omega_{\Lambda})^{1/6} \hbar c}{E^*(N)},$$

$$\text{c)} \quad E^*(N) = \frac{E_{\text{exc}}(N)}{N 2\pi} = \frac{(\Omega_{\text{dm}} + \Omega_{\Lambda})^{1/6} E_{\text{BB}}}{N 2\pi}. \tag{6.5}$$

Then, for the cases $N = 3, 6, 12$, Eq. (6.5 c) yields

$$\text{a)} \quad E^*(3) = (\Omega_{\text{dm}} + \Omega_{\Lambda})^{1/6} E_{\text{BB}} / 6\pi = 5.017 \cdot 10^{89} \text{ eV},$$

$$\text{b)} \quad E^*(6) = (\Omega_{\text{dm}} + \Omega_{\Lambda})^{1/6} E_{\text{BB}} / 12\pi = 2.509 \cdot 10^{89} \text{ eV},$$

$$\text{c)} \quad E^*(12) = (\Omega_{\text{dm}} + \Omega_{\Lambda})^{1/6} E_{\text{BB}} / 24\pi = 1.254 \cdot 10^{89} \text{ eV}. \tag{6.6}$$

Thus, the cases $N = 3$ and $N = 6$ are described by the energies (6.6 a) and (6.6 b), which agree with dark energy $E_{\text{d}} = 5.017 \cdot 10^{89}$ eV (see Eq. (3.10) of Ref. [1]) and with the half dark energy $\frac{1}{2} E_{\text{d}} = 2.509 \cdot 10^{89}$ eV (see Eqs. (3.11) and (3.12) of Ref. [1]) in the massive universe, respectively. The case $N = 12$, which is defined by the doubling of the number $N = 6$ of the sterile neutrino-antineutrino pairs (see above), is taken into account by the energy (6.6 c). Therefore, the energy (6.6 c) must be identical with $\frac{1}{4} E_{\text{d}} = 1.254 \cdot 10^{89}$ eV, which plays an important role in Ref. [1].

In Ref. [1], this dark energy $E_{\text{d}} = 5.017 \cdot 10^{89}$ eV was calculated on a complete other way.

By Eqs. (6.5 b) and (6.6), we find the corresponding wavelength λ to

$$\text{a)}\quad \lambda = \frac{(\Omega_{dm}+\Omega_\Lambda)^{1/6}\hbar c}{E^*(3)} = \frac{\hbar c}{E_{BB}/6\pi} = 3.90\cdot 10^{-95}\ \text{cm},$$

$$\text{b)}\quad \lambda = \frac{(\Omega_{dm}+\Omega_\Lambda)^{1/6}\hbar c}{E^*(6)} = \frac{\hbar c}{E_{BB}/12\pi} = 7.80\cdot 10^{-95}\ \text{cm},$$

$$\text{c)}\quad \lambda = \frac{(\Omega_{dm}+\Omega_\Lambda)^{1/6}\hbar c}{E^*(12)} = \frac{\hbar c}{E_{BB}/24\pi} = 1.56\cdot 10^{-94}\ \text{cm}. \qquad (6.7)$$

Introducing the reduced wavelength $\lambda\!\!\!^{-} = \lambda/2\pi$, Eq. (6.7) provides

$$\text{a)}\quad \lambda\!\!\!^{-} = \frac{(\Omega_{dm}+\Omega_\Lambda)^{1/6}\hbar c}{2\pi E^*(3)} = \frac{\hbar c}{E_{BB}/3} = 6.21\cdot 10^{-96}\ \text{cm},$$

$$\text{b)}\quad \lambda\!\!\!^{-} = \frac{(\Omega_{dm}+\Omega_\Lambda)^{1/6}\hbar c}{2\pi E^*(6)} = \frac{\hbar c}{E_{BB}/6} = 1.24\cdot 10^{-95}\ \text{cm},$$

$$\text{c)}\quad \lambda\!\!\!^{-} = \frac{(\Omega_{dm}+\Omega_\Lambda)^{1/6}\hbar c}{2\pi E^*(12)} = \frac{\hbar c}{E_{BB}/12} = 2.48\cdot 10^{-95}\ \text{cm}. \qquad (6.8)$$

By generalization of Eq. (6.8), we get $R_{BB} = \lambda\!\!\!^{-}/N = \hbar c/E_{BB}$, where N describes again the sterile neutrino-antineutrino pair number, i.e. we have $\lambda\!\!\!^{-} = NR_{BB}$ because of N as the basis of the zero-point oscillations.

Consequently, for example, in the case $N=1$, we have $\lambda\!\!\!^{-} = R_{BB} = \hbar c/E_{BB}$, i.e. according to Eq. (3.135) of Ref. [1] this one sterile neutrino-antineutrino pair yields its decay in one photon and one graviton at $R_{BB} = 2.069\cdot 10^{-96}$ cm as the beginning of the quantum universe (see Eqs. (3.8) or (5.10)).

Therefore, generally, according to Ref. [1], the particle-antiparticle pair number N is given by

$$N = \frac{R}{R_{BB}}, \qquad (6.9)$$

where R describes again the scale factor, which is defined by Eq. (4.15). Thus, generally, i.e. also for the classical universe (anti-universe), as shown in Ref. [1], the particle (antiparticle) number $\tilde{N}$ is equivalent to

$$\widetilde{N} = \tfrac{1}{2} N = \frac{\tfrac{1}{2} R}{R_{\mathrm{BB}}}. \tag{6.10}$$

Generally, using the data of Tables II and IV, the corresponding results, derived in this Sec. 6 for the universe, are also valid for the anti-universe.

7 The cosmological inflation

In this chapter, we show that at the Planck length the cosmological inflation event [1-6] of the strong-electroweak symmetry breaking is plausible.

Firstly, we assume that a cosmological inflation event can be taken place at the critical temperature $T_{\mathrm{H}^0} = E_0(\mathrm{H}^0)/k \approx 1.46 \cdot 10^{15}\ \mathrm{K}$ of the Higgs field [12] in the electromagnetic-weak symmetry breaking, where $E_0(\mathrm{H}^0) \cong 126\ \mathrm{GeV}$ [17] and k describe the rest energy of the Higgs boson and the Boltzmann constant, respectively. Consequently, according to Ref. [6], for the radiation-dominated classical universe, the beginning of the inflation (i) is determined by

$$t_{\mathrm{i}} \approx \frac{3.25936}{\sqrt{N(T)}} \left(\frac{10^{10}\ \mathrm{K}}{T} \right)^2 \approx 1.56 \cdot 10^{-11}\ \mathrm{s}, \tag{7.1}$$

where $N(T) = 385/4$ [1].

Then, using $R \propto t^{1/2}$ (see Eq. (4.26)) of this radiation-dominated universe and its beginning at $\widetilde{R} \approx R_{\mathrm{Pl}}$ (see Eq. (4.25)), the scale factor R_{i} of the start of the inflation can be calculated by

$$R_{\mathrm{i}} \approx R_{\mathrm{Pl}} \cdot \sqrt{\frac{t_{\mathrm{i}}}{t_{\mathrm{Pl}}}} \approx 2.75 \cdot 10^{-19}\ \mathrm{m}, \tag{7.2}$$

where R_{Pl} and t_{Pl} describe the Planck length and the Planck time, respectively.

By $T_0 = 2.7255\ \mathrm{K}$ (see Table III), we obtain

$$1+z_{\mathrm{H}^0}=\frac{T_{\mathrm{H}^0}}{T_0}\approx 5.36\cdot 10^{14}\,, \tag{7.3}$$

so that the end (e) of the inflation takes place at

$$R_{\mathrm{e}}=\frac{R_0}{1+z_{\mathrm{H}^0}}\approx 2.57\cdot 10^{11}\,\mathrm{m}, \tag{7.4}$$

where $R_0=1.375\cdot 10^{26}\,\mathrm{m}$ defines the Hubble length (see Eq. (5.4) of Ref. [6]).

Therefore, via the inflation, the scale factor of the classical universe enlarges by the enormous factor

$$\frac{R_{\mathrm{e}}}{R_{\mathrm{i}}}\approx 9.35\cdot 10^{29}\,. \tag{7.5}$$

Using $R\propto t^{2/3}$ (see Eq. (4.31)) for the matter-dominated classical universe, we find the scale factor R_{dec} of the decoupling (dec) of the photons at $t_{\mathrm{dec}}=3.72\cdot 10^5\,\mathrm{yr}$ [8] to

$$R_{\mathrm{dec}}\approx R_0\left(\frac{t_{\mathrm{dec}}}{t_0}\right)^{2/3}\approx 1.236\cdot 10^{23}\,\mathrm{m}, \tag{7.6}$$

where $t_0=13.81\,\mathrm{Gyr}$ describes the age of the present classical universe (see Table III). Then, during the radiation-dominated epoch of the classical universe (see above), we have

$$t_{\mathrm{e}}\approx t_{\mathrm{dec}}\cdot\left(\frac{R_{\mathrm{e}}}{R_{\mathrm{dec}}}\right)^2\approx 5.08\cdot 10^{-11}\,\mathrm{s}\,, \tag{7.7}$$

i.e. the event of the inflation pasts rapidly.

Secondly, the same scenario can be assumed at a critical temperature $T_{\mathrm{M}}=E_0(\mathrm{M})/k\approx 7.948\cdot 10^{30}\,\mathrm{K}$ of the monopole field in the strong-electroweak symmetry breaking, where $E_0(\mathrm{M})\cong 6.849\cdot 10^{17}\,\mathrm{GeV}$ [6] describes the rest energy of the magnetic monopoles.

Consequently, if we assume here $N(T)=1/2\,\Omega_\gamma$ (see e.g., Eq. [4.22) in this work or Eq. (5.14) in Ref. [6] for $t=t_{\mathrm{Pl}}$), the beginning of the inflation is determined by

$$t_{\mathrm{i}} \cong \frac{3.25936}{\sqrt{N(T)}}\left(\frac{10^{10}\,\mathrm{K}}{T_{\mathrm{M}}}\right)^{2} \cong t_{\mathrm{Pl}} \cong 5.391\cdot 10^{-44}\,\mathrm{s}\,, \tag{7.8}$$

where $\Omega_{\gamma} = 5.46\cdot 10^{-5}$ describes the radiation density parameter (see Table III). Then, because of $R \propto t^{1/2}$ and the beginning of the radiation-dominated classical universe at $R \approx R_{\mathrm{Pl}}$, the scale factor R_{i} of the start of the inflation can be calculated by

$$R_{\mathrm{i}} \cong R_{\mathrm{Pl}} \cdot \sqrt{\frac{t_{\mathrm{i}}}{t_{\mathrm{Pl}}}} \cong R_{\mathrm{Pl}} \cong 1.6162\cdot 10^{-35}\,\mathrm{m}. \tag{7.9}$$

Analogously to (7.3), we obtain

$$1 + z_{\mathrm{M}} = \frac{T_{\mathrm{M}}}{T_0} \approx 2.916\cdot 10^{30}\,, \tag{7.10}$$

so that the end of the inflation takes place here at

$$R_{\mathrm{e}} = \frac{R_0}{1+z_{\mathrm{M}}} = \widetilde{R}_0 \approx 4.713\cdot 10^{-5}\,\mathrm{m}, \tag{7.11}$$

where $\widetilde{R}_0$ was introduced first in Ref. [6].

Therefore, via the inflation, the scale factor of the classical universe enlarges by the enormous factor

$$\frac{R_{\mathrm{e}}}{R_{\mathrm{i}}} = \frac{\widetilde{R}_0}{R_{\mathrm{Pl}}} \approx 2.916\cdot 10^{30}\,. \tag{7.12}$$

Considering the inflationary phase $\Delta t = t_{\mathrm{e}} - t_{\mathrm{i}} = t_{\mathrm{X,Y}} - t_{\mathrm{Pl}} \cong t_{\mathrm{X,Y}}$ {see Eqs. (5.23) and (5.24) of Ref. [6]}, the result (5.24) of Ref. [6] yields for the end of the inflationary phase

$$t_{\mathrm{e}} \approx t_{\mathrm{X,Y}} \approx \frac{\ln(1+z_{\mathrm{M}})}{H_{\mathrm{V}}} \approx 2.7\cdot 10^{-40}\,\mathrm{s}\,, \tag{7.13}$$

i.e. the event of the inflation pasts rapidly in accordance with Eq. (4.6) of Ref. [6], whereat the quantity $H_{\mathrm{V}} = 2.63\cdot 10^{41}\,\mathrm{s}^{-1}$ describes the vacuum Hubble constant [6].

Therefore, using $R \propto t^{2/3}$ for the matter-dominated classical universe (see above) and Eq. (5.16) of Ref. 6, we can assume the scale factor R_{dec} of the decoupling of the photons (see above) to

$$R_{\mathrm{dec}} \approx \frac{R_0}{\sqrt{N_{\mathrm{X,Y}}(T)}} \left(\frac{t_{\mathrm{dec}}}{t_0} \right)^{2/3} \approx 9.8 \cdot 10^{21}\,\mathrm{m}, \tag{7.14}$$

where $N_{\mathrm{X,Y}}(T) \equiv 160.75$ [6, 11]. Then, during the radiation-dominated epoch of the universe, in accordance with the result (7.13), we have

$$t_{\mathrm{e}} \approx t_{\mathrm{dec}} \cdot \left(\frac{R_{\mathrm{e}}}{R_{\mathrm{dec}}} \right)^2 \approx 2.7 \cdot 10^{-40}\,\mathrm{s}, \tag{7.15}$$

i.e. the event of the inflation pasts again rapidly.

We assume that the second scenario (7.8) to (7.15) is correct. This assumption is supported by the strong-electroweak symmetry breaking at the Planck distance $R_{\mathrm{Pl}} = 1.6162 \cdot 10^{-35}\,\mathrm{m}$, which via the great gauge group yields the physics of the SUSY GUTs [8]. Generally, the reason of this assumption is that via $E_0(\mathrm{X}) = E_0(\mathrm{Y}) = 2.675 \cdot 10^{16}\,\mathrm{GeV}$ of the X and Y gauge bosons [6] the coupling constant of the SUSY GUTs was found to

$$\alpha_{\mathrm{GUT}} = \frac{E_0(\mathrm{X})}{E_0(\mathrm{M})} = 0.0391 \tag{7.16}$$

in excellent agreement with the value $\alpha_{\mathrm{GUT}} \approx 0.04$ obtained by a completely different way, using the extrapolation of the gauge couplings constants of the standard model to very high energies [6]. Additionally, as reasonable argument, we have

$$\widetilde{R}_0 = (1 + z_{\mathrm{M}})\, R_{\mathrm{Pl}} = 4.713 \cdot 10^{-5}\,\mathrm{m} \tag{7.17}$$

as the end of the early classical universe [6], whereas the expression

$$R_0 = (1 + z_{\mathrm{M}})\, \widetilde{R}_0 = 1.375 \cdot 10^{26}\,\mathrm{m} \tag{7.18}$$

yields the scale factor of the present classical universe for $z = 0$ [6], so that the early classical universe ($R_{\mathrm{Pl}} \le R \le \widetilde{R}_0$) enlarges by the enormous factor (7.10) to the late classical universe ($\widetilde{R}_0 \le R \le R_0$).

Besides, the hitherto existing cosmological models [7-9] give no reason for the assumption of two cosmological inflation events.

Using the data of Tables II and IV, all results of Sec. 7, derived for the classical universe, are also valid for the classical anti-universe.

8 Several cosmic energy considerations in connection with a well-established prediction of the quantum field theory

In this chapter, all considerations, which are performed for the total (quantum and classical) universe, are also valid for the corresponding anti-universe, when we use the results of the Tables II and IV.

Using Eqs. (3.1) and (3.3), for the quantum universe, we find the vacuum energy density to

$$\widehat{\rho}_{\mathrm{vac}}\, c^2 = \widehat{\rho}_{\mathrm{vac}}(T)\, c^2 = \frac{2}{3}\Omega_\gamma \frac{\pi^2}{15} \frac{E_{\mathrm{Pl}}^6}{(\hbar c)^3\, (kT)^2}. \tag{8.1}$$

For the final state of the classical universe, by Eq. (3.75 a) of Ref. [1], we have derived the dark energy density to

$$\rho_{\mathrm{f1}} c^2 = \Omega_\Lambda \rho_{0\mathrm{C}} c^2 \left(\frac{d_{\mathrm{eff}}}{c\, t_{\mathrm{f2}}}\right)^3 \left(\frac{3\, N(T)}{2}\right)^{3/4} = 4.56 \cdot 10^{-4}\ \mathrm{eV\,cm^{-3}}, \tag{8.2}$$

where $\Omega_\Lambda = 0.685$ (see Table III), $\rho_{0\mathrm{C}}\, c^2 = 4.77 \cdot 10^3\ \mathrm{eV\,cm^{-3}}$ (see Table III), $d_{\mathrm{eff}} = 3.321 \cdot 10^{28}$ cm (see Eq. (3.8) of Ref. [1]), $t_{\mathrm{f2}} = 2.172 \cdot 10^{20}$ s (see Eq. (3.36) of Ref. [1]) and $N(T) = (4/11)^{1/3}$ (see Eq. (3.73) of Ref. [1]) describe the dark energy density parameter, the present critical energy density, the effective distance of the approximate end of the present accelerated expansion, the end time of the final state of the classical universe and the statistical value, respectively. After the end of the accelerated expansion, i.e. approximately from the distance d_{eff}, where we get again a separation of the space-time, we have a linear expansion [1, 2], which takes place in a three-dimensional space.

If we set $\widehat{\rho}_{\mathrm{vac}} = \rho_{\mathrm{f1}}$, we obtain as solution

$$k\widetilde{T}_{\mathrm{BB}} = \widetilde{E}_{\mathrm{BB}} = \left[\frac{2}{3}\Omega_\gamma \frac{\pi^2}{15}\frac{E_{\mathrm{Pl}}^6}{(\hbar c)^3 \rho_{\mathrm{fl}} c^2}\right]^{1/2} \cong 4.76\cdot 10^{90}\ \mathrm{eV} \qquad (8.3)$$

in accordance with the expression (3.11). Because of $\widehat{\rho}_{\mathrm{vac}} = \rho_{\mathrm{fl}}$, the result (8.3) must be valid not only for the beginning of the quantum universe but also for the final state of the classical universe, so that their temperature is given by

$$\widetilde{T}_{\mathrm{BB}} = \widetilde{E}_{\mathrm{BB}}/k \cong 5.52\cdot 10^{94}\ \mathrm{K}. \qquad (8.4)$$

This heat accumulation (8.4) is a consequence of the total particle number $N_{\mathrm{BB}} = \widetilde{T}_{\mathrm{BB}}/T_{\mathrm{Pl}} = 3.90\cdot 10^{62}$ (see also Eq. (9.3)), since the Planck temperature $T_{\mathrm{Pl}} = 1.417\cdot 10^{32}\,\mathrm{K}$ is the greatest possible particle temperature, whereas the smallest possible particle temperature is given by

$$T_\gamma = E_0(\gamma)/k = T_{\mathrm{G}} = E_0(\mathrm{G})/k = 1.814\cdot 10^{-31}\,\mathrm{K}. \qquad (8.5)$$

In contrast to the result (8.3), by Eq. (3.10) of Ref. [1], for the final state of the classical universe, we have found only the dark (d) energy

$$E_{\mathrm{d}} = \Omega_\Lambda \rho_{0\mathrm{C}} c^2 \tfrac{4}{3}\pi d_{\mathrm{eff}}^3 \cong 5.02\cdot 10^{89}\ \mathrm{eV}. \qquad (8.6)$$

This contradiction can be explained as follows. Taking the parameters of Table III, we can calculate the Hubble energy to

$$E_{\mathrm{H}} \cong \rho_{0\mathrm{C}} c^2 \tfrac{4}{3}\pi R_0^3 \cong 5.194\cdot 10^{88}\ \mathrm{eV}. \qquad (8.7)$$

Then, considering the mass of the classical universe from $R_0 = 1.375\cdot 10^{28}$ cm (see Table III) to $d_{\mathrm{eff}} = 3.321\cdot 10^{28}$ cm (see above), we obtain a total energy E_{tot}^* of the final state of the classical universe to

$$E_{\mathrm{tot}}^* = \frac{E_{\mathrm{d}}}{E_{\mathrm{H}}} E_{\mathrm{d}} \cong 4.85\cdot 10^{90}\ \mathrm{eV}. \qquad (8.8)$$

This total energy, which contains the dark matter $\Omega_{\mathrm{dm}} = 0.265$ (see Table III) and the dark energy $\Omega_\Lambda = 0.665$ (see Table III), is thus determined by the 3 sterile neutrino types $\hat{\nu}_\Lambda$, $\hat{\nu}_{\mathrm{dm}}$ and $\hat{\nu}_{\mathrm{b}}$ [1, 2]. In this work, it is usual to consider the energy for one particle type. Thus, the total energy (8.8) must be multiplied by $(\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/3}$ as square factor $(\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/6} \times (\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/6}$ of their two decay products (see Ref. [1]), i.e. we get the reduced total energy

$$E_{\text{tot}}^{**} = (\Omega_{\text{dm}} + \Omega_{\Lambda})^{1/3} E_{\text{tot}}^{*} \cong 4.768 \cdot 10^{90}\ \text{eV}, \tag{8.9}$$

which agrees excellently with the result (8.3).

According to Eq. (3.135) of Ref. [1], in the final state of the classical universe, we have shown that one sterile neutrino type pair decays into two photons and two gravitons, i.e. we have effectively one photon and one graviton per one sterile neutrino type.

This fact was considered by the two decay products of the sterile neutrinos in Sec. 3.1 of Ref. [1].

Thus, for the classical universe, at the Planck time $t_{\text{Pl}} = 5.39106 \cdot 10^{-44}\ \text{s}$, we obtain the Planck energy $E_{\text{Pl}} = \hbar / t_{\text{Pl}}$ as geometric mean of

$$\sqrt{[E_0(\gamma) + E_0(\text{G})] E_{\text{tot}}^{**}} = E_{\text{Pl}} = \hbar / t_{\text{Pl}} = 1.2209 \cdot 10^{28}\ \text{eV}. \tag{8.10}$$

Similarly, for the quantum universe, at $t_{\text{Pl}} = 5.39106 \cdot 10^{-44}\ \text{s}$, we obtain the Planck energy $E_{\text{Pl}} = \hbar / t_{\text{Pl}}$ as geometric mean of

$$\sqrt{[E_0(\gamma) + E_0(\text{G})] \widetilde{E}_{\text{BB}}} = E_{\text{Pl}} = \hbar / t_{\text{Pl}} = 1.2209 \cdot 10^{28}\ \text{eV}, \tag{8.11}$$

where $\widetilde{E}_{\text{BB}}$ is given by Eq. (3.11).

Consequently, at the Planck time $t_{\text{Pl}} = 5.39106 \cdot 10^{-44}\ \text{s}$, every particle has the Planck energy as its greatest possible relativistic energy.

Therefore, for the total universe, we must take into account either E_{tot}^{**} or $\widetilde{E}_{\text{BB}}$, so that the results (8.10) and (8.11) confirm the assumption

$$E_{\text{tot}}^{**} = \widetilde{E}_{\text{BB}} \cong 4.769 \cdot 10^{90}\ \text{eV}. \tag{8.12}$$

Thus, for extremely relativistic photons and gravitons we can apply the Planck number density

$$n_{\text{Pl}} = 20.286843 \left(\frac{T_{\text{Pl}}}{\text{K}} \right)^3 \text{cm}^{-3} = 5.770 \cdot 10^{97}\ \text{cm}^{-3}, \tag{8.13}$$

where the Planck temperature $T_{\text{Pl}} = 1.416833 \cdot 10^{32}\ \text{K}$ is the mean temperature of the photons and gravitons. Then, the thermal Planck energy density is given by

$$\rho_{\text{Pl}}(T_{\text{Pl}})\, c^2 = E_{\text{Pl}}\, n_{\text{Pl}} = 7.045 \cdot 10^{125}\ \text{eV cm}^{-3}. \tag{8.14}$$

For the quantum universe, by the result (3.14), we can form the energy density

$$\left[E_0(\gamma)+E_0(\mathrm{G})\right]n_{\mathrm{P}}(R_{\mathrm{BB}})=2\,\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\,c^2\cong 8.453\cdot 10^{247}\ \mathrm{eV}. \qquad (8.15)$$

In Eq. (8.15), the result $2\,\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\,c^2$ is understandable, since we have considered only the photon of the two decay products of the sterile neutrinos at the derivation of the parameters of the big bang (see, e.g., Eq. (3.22) of Ref. [1]), so that we must take into account the factor $2=\left[E_0(\gamma)+E_0(\mathrm{G})\right]/E_0(\gamma)$ for Eq. (3.12) because of the photon and the graviton as the 2 decay products of one sterile neutrino type (see above), which were not considered simultaneously at the Eqs. (3.12 to (3.15).

Using Eqs. (8.14) and (8.15) as well as (3.12) and (9.4), we can introduce

$$\frac{\left[E_0(\gamma)+E_0(\mathrm{G})\right]n_{\mathrm{P}}(R_{\mathrm{BB}})}{\rho_{\mathrm{Pl}}(T_{\mathrm{Pl}})\,c^2}=\frac{\rho_{\mathrm{vac}}(R_{\mathrm{BB}})\,c^2}{\frac{1}{2}E_{\mathrm{Pl}}n_{\mathrm{Pl}}}\cong 1.2\cdot 10^{122}\ , \qquad (8.16)$$

so that the thermal Planck energy density (8.14) must increase to obtain the level to the beginning of the quantum universe by a factor of about 10^{122}.

This result (8.16) can be confirmed as follows. In Ref. [1], we have shown that the gravitation determines the evolution of the total universe via the kinetic energy $\overline{E}_{\mathrm{max}}=2.51\cdot 10^{212}$ eV (see Eq. (3.113) of Ref. [1]) from its beginning to its end, whereat simultaneously the potential energy $\overline{E}_{\mathrm{max}}=2.51\cdot 10^{212}$ eV (see Eq. (3.134) of Ref. [1]), which is necessary for the transition from its final state to its beginning, is formed in its final state.

Thus, during this evolution of the universe, from its beginning to its final state, it is produced the total energy

$$2\,\overline{E}_{\mathrm{max}}=5.02\cdot 10^{212}\ \mathrm{eV}. \qquad (8.17)$$

Thus, during this evolution of the universe, the energy $\widetilde{E}_{\mathrm{BB}}=4.769\cdot 10^{90}$ eV (see Eq. (3.11)), which determine the beginning of the quantum universe, must be provided about 10^{122} times because of

$$\frac{2\,\overline{E}_{\mathrm{max}}}{\widetilde{E}_{\mathrm{BB}}}\cong 1.1\cdot 10^{122}. \qquad (8.18)$$

The results (8.16) and (8.18) agree excellently with the expression (5.7), which yields

$$\frac{(\Lambda_{BB}\,\Lambda_\Lambda)^{1/2}}{\Lambda_\Lambda} = (\Lambda_{BB}/\Lambda_\Lambda)^{1/2} \cong 1.1\cdot 10^{122}\,, \tag{8.19}$$

where we have simulated a continuous evolution for the quantum and classical universe because of the inflation via the time-dependent cosmological constants Λ_{BB} [for the big bang of the quantum universe (see Eq. (5.4))] and Λ_Λ [for the present dark energy of the classical universe (see Eq. (5.2))] by $(\Lambda_{BB}\,\Lambda_\Lambda)^{1/2}$.

Then, using correspondingly the vacuum energy density $\rho_{vac}(R_{BB})c^2 \cong 4.227\cdot 10^{247}\ \mathrm{eV\,cm^{-3}}$ (see Eq. (3.12)) and the present dark energy density $\rho_\Lambda c^2 = \Omega_\Lambda\,\rho_{0C}\,c^2 = 3.27\cdot 10^3\ \mathrm{eV\,cm^{-3}}$ (see Table III), we can also write

$$\frac{(\rho_{vac}(R_{BB})\,\rho_\Lambda)^{1/2}}{\rho_\Lambda} = (\rho_{vac}(R_{BB})/\rho_\Lambda)^{1/2} \cong 1.1\cdot 10^{122}\,, \tag{8.20}$$

i.e. the mean density $(\rho_{vac}(R_{BB})\,\rho_\Lambda)^{1/2} = 3.72\cdot 10^{125}\ (\mathrm{eV}/c^2)\,\mathrm{cm^{-3}}$, which is necessary because of the inflation, is a factor of about 10^{122} greater than the present vacuum density ρ_Λ.

In Eq. (8.20), at $\rho_{vac}(R_{BB})$, the factor 2 (see, e.g., Eq. (8.15)) is not necessary, since the 2 decay products (photon and graviton) of one sterile neutrino type (see above) are included in ρ_Λ (see, e.g., Sec. 3.1 of Ref. [1]).

Because of the geometric mean of the results (5.5) and (5.6), we can assume the mean density $(\rho_{vac}(R_{BB})\,\rho_\Lambda)^{1/2}$ as a reasonable density at the Planck scale, so that we have confirmed the proposition of the quantum field theory, which predicts a factor of about 10^{122} between the vacuum density at the Planck scale and the present vacuum density ρ_Λ.

More generally, the results (8.16) and (8.18) support the conclusions (5.7) as well as (8.19) and (8.20).

Correspondingly, the half vacuum energy density (4.23) {see also Refs. [1-4]}, divided by the photon energy density

$$\rho_0(\gamma)c^2 = \frac{\pi^2}{15}\frac{(kT_0)^4}{(\hbar c)^3} = 0.2606\ \text{eV cm}^{-3} \quad \text{(see Eq. (4.9))}, \tag{8.21}$$

yields also the factor of about 10^{122} because of

$$\frac{\frac{1}{2}\rho_{\text{vac}}(R_{\text{Pl}})}{\rho_0(\gamma)} \cong 1.3 \cdot 10^{122}. \tag{8.22}$$

Consequently, we have explained in detail the significance of the factor of about 10^{122}, predicted by the quantum field theory because of the result (9.4) via the half Planck energy density (8.14) and $\rho_\Lambda c^2$ (see above) according to

$$\frac{\frac{1}{2}\rho_{\text{Pl}}(T_{\text{Pl}})}{\rho_\Lambda} = \frac{\frac{1}{2}E_{\text{Pl}}\,n_{\text{Pl}}}{\rho_\Lambda c^2} \cong 1.1 \cdot 10^{122}. \tag{8.23}$$

9 Justification of the new thermal equilibrium via the massive particle formation in the quantum universe (anti-universe)

In contrast to the General Relativity, because of the four-dimensional quantum gravity [1-3], in the quantum universe (anti-universe), at the massive particle formation, two facts must be taken into account.

Firstly, this new quantum gravity [3], by which the quantum field theory and the General Relativity are unified as shown in Ref. [1] via the time-dependent vacuum energy density or cosmological constant, leads to a quantization of the continuity of the space and the time in the ranges of the space distances $R_{\text{BB}} \le R \le R_{\text{Pl}}$ and the time differences $t_{\text{BB}} = R_{\text{BB}}/c \le t = R/c \le t_{\text{Pl}} = R_{\text{Pl}}/c$. This quantization means again a separation of the space-time into its quantized components of the three-dimensional gravitational space $R = |\pm R|$ and the one-dimensional time because of the space-distance-dependent time $t = R/c$, since this time is no longer a space-independent coordinate as the Planck time proves. Therefore, in the quantum universe and anti-universe, because of $R = |\pm R|$ and $t = \pm t$, the CPT invariance (see also Sec. 2) is valid, whereas we have only the time reversal (see also Ref. [1]) between the classical universe

($t=+t$) and anti-universe ($t=-t$) at positive four-dimensional scale factors R.

Secondly, according to Ref. [3], between the quantum universe and anti-universe, the gravitation leads to the separation of their photon(particle)-photon(antiparticle) pairs of the virtual matter (quantum vacuum) into photon(particle) pairs as matter (universe) and photon(antiparticle) pairs as antimatter (anti-universe), so that we have always photon and particle pairs as well as photon and antiparticle pairs in the quantum universe and anti-universe, respectively. Because of the transition from the quantum universe to the classical universe this pairing result is transferable to the classical universe.

Therefore, at the Planck time, for the quantum universe, instead of the result (8.11), we can also introduce

$$\sqrt{2E_0(\gamma)\times\widetilde{E}_{\mathrm{BB}}}=E_{\mathrm{Pl}}=\hbar/t_{\mathrm{Pl}}=1.2209\cdot10^{28}\ \mathrm{eV}, \tag{9.1}$$

so that we can assume that every particle has the (mean) Planck energy E_{Pl} as greatest possible relativistic energy.

Instead of Eq. (9.1), we can also write

$$\frac{\widetilde{E}_{\mathrm{BB}}}{E_{\mathrm{Pl}}}=\frac{E_{\mathrm{Pl}}}{2E_0(\gamma)}=\frac{\frac{1}{2}E_{\mathrm{Pl}}}{E_0(\gamma)}=3.906\cdot10^{62}, \tag{9.2}$$

i.e. the left term

$$\frac{\widetilde{E}_{\mathrm{BB}}}{E_{\mathrm{Pl}}}=3.906\cdot10^{62} \tag{9.3}$$

describes the number of all particles, which have the Planck energy as greatest possible relativistic energy, whereas because of the uncertainty relation

$$\frac{1}{2}E_{\mathrm{Pl}}=\frac{\hbar}{2t_{\mathrm{Pl}}} \tag{9.4}$$

the right term

$$\frac{\frac{1}{2}E_{\mathrm{Pl}}}{E_0(\gamma)}=N_\gamma=3.906\cdot10^{62} \tag{9.5}$$

gives the number of all photons, which determines the half Planck energy as their greatest possible kinetic energy.

Analogous to Eq. (9.5), for example, we can assume the number of all protons and magnetic monopoles to

$$\frac{\frac{1}{2}E_{\mathrm{Pl}}}{E_0(\mathrm{p})} = N_{\mathrm{p}} = 6.506 \cdot 10^{18} \tag{9.6}$$

and

$$\frac{\frac{1}{2}E_{\mathrm{Pl}}}{E_0(\mathrm{M})} = N_{\mathrm{M}} = 8.913\,. \tag{9.7}$$

Thus, because of the results (9.5) to (9.7), these particles must possess approximately the speed of light

$$c_\gamma = \left[1 - \left(\frac{E_0(\gamma)}{\frac{1}{2}E_{\mathrm{Pl}}}\right)^2\right] c = (1 - 6.555 \cdot 10^{-126})\, c\,, \tag{9.8}$$

$$c_{\mathrm{p}} = \left[1 - \left(\frac{E_0(\mathrm{p})}{\frac{1}{2}E_{\mathrm{Pl}}}\right)^2\right] c = (1 - 2.362 \cdot 10^{-38})\, c \tag{9.9}$$

and

$$c_{\mathrm{M}} = \left[1 - \left(\frac{E_0(\mathrm{M})}{\frac{1}{2}E_{\mathrm{Pl}}}\right)^2\right] c = (1 - 1.259 \cdot 10^{-2})\, c\,. \tag{9.10}$$

By the new thermal equilibrium [1-6], the photon energy kT is equivalent to the particle (P) rest energy $E_0(\mathrm{P})$, i.e. we have assumed

$$kT = E_0(\mathrm{P})\,. \tag{9.11}$$

Because of Eq. (9.11), we must assume that the massive particles are formed by the reduction of the kinetic energy of the photons as a result of a decrease of their velocity.

Consequently, instead of the expressions (9.5) to (9.10), we can assume the conditions

a) $E_0(\gamma) \to E_0(\mathrm{p})\,,$ b) $E_0(\mathrm{p}) \to \frac{1}{2}E_{\mathrm{Pl}}\,,$

$c_\gamma^* = (1 - 2.775 \cdot 10^{-88})\, c\,,$ $c_{\mathrm{p}} = (1 - 2.362 \cdot 10^{-38})\, c\,,$

$\dfrac{c - c_\gamma^*}{c} = 2.775 \cdot 10^{-88}\,,$ $\dfrac{c - c_{\mathrm{p}}}{c} = 2.362 \cdot 10^{-38}\,,$

c) $\dfrac{c - c_\gamma^*}{c} \times \dfrac{c - c_{\mathrm{p}}}{c} = 6.555 \cdot 10^{-126}$ (see Eq. (9.8)) (9.12)

and

$$\text{a)}\quad E_0(\gamma)\to E_0(\mathrm{M}),\qquad c_\gamma^{**}=(1-5.208\cdot10^{-124})\,c,\qquad \frac{c-c_\gamma^{**}}{c}=5.208\cdot10^{-124},$$

$$\text{b)}\quad E_0(\mathrm{M})\to \tfrac{1}{2}E_{\mathrm{Pl}},\qquad c_{\mathrm{M}}=(1-1.259\cdot10^{-2})\,c,\qquad \frac{c-c_{\mathrm{M}}}{c}=1.259\cdot10^{-2},$$

$$\text{c)}\quad \frac{c-c_\gamma^{**}}{c}\times\frac{c-c_{\mathrm{p}}}{c}=6.555\cdot10^{-126}\quad(\text{see Eq. (9.8)}). \tag{9.13}$$

Indeed, the results (9.12) and (9.13) show precisely and uniquely that by photons the formation of the massive particles is favored energetically in comparison with the direct transition $E_0(\gamma)\to \tfrac{1}{2}E_{\mathrm{Pl}}$ (see Eq. (9.8)). We see also that this process of the massive particle formation is most favorable for small massive rest energies.

Using the data of Tables II and IV, all results, derived in this Sec. 9 for the universe, are also valid for the anti-universe.

10 The particle energy densities of the quantum and classical universe (anti-universe)

For example, at protons, via their distance (3.22) and their twofold vacuum energy density (3.24), when we consider the plausible factor $(R_{\mathrm{p}}/R_{\mathrm{Pl}})^3$ and the proton number (9.6), we obtain again the vacuum energy density (3.18) or (4.23) at the Planck scale according to

$$\rho_{\mathrm{vac}}(R_{\mathrm{Pl}})c^2=$$
$$=2\,\rho_{\mathrm{vac}}(R_{\mathrm{p}})c^2N_{\mathrm{p}}\times(R_{\mathrm{p}}/R_{\mathrm{Pl}})^3\cong6.93\cdot10^{121}\,\mathrm{eV\,cm^{-3}}. \tag{10.1}$$

Then, when we use therefore again the vacuum energy density (3.18), divided by the particle numbers (9.5) to (9.7), for the quantum universe, we obtain the particle energy densities

$$\rho_\gamma(R_{\mathrm{Pl}})c^2=\frac{\rho_{\mathrm{vac}}(R_{\mathrm{Pl}})c^2}{N_\gamma}=1.774\cdot10^{59}\,\mathrm{eV\,cm^{-3}}, \tag{10.2}$$

$$\rho_{\mathrm{p}}(R_{\mathrm{Pl}})c^2 = \frac{\rho_{\mathrm{vac}}(R_{\mathrm{Pl}})c^2}{N_{\mathrm{p}}} = 1.065 \cdot 10^{103}\ \mathrm{eV\,cm^{-3}} \tag{10.3}$$

and

$$\rho_{\mathrm{M}}(R_{\mathrm{Pl}})c^2 = \frac{\rho_{\mathrm{vac}}(R_{\mathrm{Pl}})c^2}{N_{\mathrm{M}}} = 7.775 \cdot 10^{120}\ \mathrm{eV\,cm^{-3}} \tag{10.4}$$

at the Planck distance R_{Pl} for photons, protons and monopoles, respectively.

However, because of the new inflation model, which is defined by the results (4.14) to (4.19), the Planck distance R_{Pl} is enlarged immediately by the enormous factor $1 + z_{\mathrm{M}} = \widetilde{R}_0 / R_{\mathrm{Pl}} = 2.916 \cdot 10^{30}$ (see Eqs. (4.16) to (4.18)), i.e. we have $\widetilde{R}_0 = (1 + z_{\mathrm{M}})\, R_{\mathrm{Pl}}$, so that the particle energy densities (10.2) to (10.4) of the quantum universe must decrease by the very small factor $1/(1 + z_{\mathrm{M}})^3$.

Thus, directly after the inflation, at the scale factor $\widetilde{R}_0$ of the early classical universe $\{(R_{\mathrm{Pl}} \le R \le \widetilde{R}_0)$ [1-6]$\}$ we obtain correspondingly the particle energy densities

$$\rho_{\gamma}(\widetilde{R}_0)c^2 = \frac{\rho_{\gamma}(R_{\mathrm{Pl}})c^2}{(1 + z_{\mathrm{M}})^3} = 7.155 \cdot 10^{-33}\ \mathrm{eV\,cm^{-3}}, \tag{10.5}$$

$$\rho_{\mathrm{p}}(\widetilde{R}_0)c^2 = \frac{\rho_{\mathrm{p}}(R_{\mathrm{Pl}})c^2}{(1 + z_{\mathrm{M}})^3} = 4.295 \cdot 10^{11}\ \mathrm{eV\,cm^{-3}} \tag{10.6}$$

and

$$\rho_{\mathrm{M}}(\widetilde{R}_0)c^2 = \frac{\rho_{\mathrm{M}}(R_{\mathrm{Pl}})c^2}{(1 + z_{\mathrm{M}})^3} = 3.135 \cdot 10^{29}\ \mathrm{eV\,cm^{-3}}. \tag{10.7}$$

Now, using Eqs. (4.1) as well as (4.3) to (4.5), we determine the particle energy densities in the early $(R_{\mathrm{Pl}} \le R \le \widetilde{R}_0)$ and the late $(\widetilde{R}_0 \le R \le R_0)$ classical universe [1-6] for photons, protons and monopoles. Attention! The scale factor $\widetilde{R}_0$ has a double role because it determines the end of the early universe and the beginning of the late universe. To this problem, we will return below. The scale factor R_0 is defined by Eq. (4.17).

For our goal, we apply additionally the results (4.25), (4.26) and (4.29). Therefore, we have

$$t = \frac{1}{2}\left(\frac{90}{8\pi^3}\right)^{1/2} \hbar E_{\mathrm{Pl}} \frac{1}{\sqrt{N(T)}\,(kT)^2} =$$

$$= \frac{(2.42035 \pm 0.00015)}{\sqrt{N(T)}}\left(\frac{\mathrm{MeV}}{kT}\right)^2 \mathrm{s}\,, \tag{10.8}$$

$$\widetilde{R} = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\,(t_{\mathrm{Pl}}\,t)^{1/2}\,, \tag{10.9}$$

$$R = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{t}{H_0}\right)^{1/2}, \tag{10.10}$$

$$\widetilde{\rho} = \frac{1}{2}N(T)\,\Omega_\gamma\,\rho_{0\mathrm{C}} \times \left(\frac{\widetilde{R}_0}{\widetilde{R}}\right)^4 \tag{10.11}$$

and

$$\rho = \frac{1}{2}N(T)\,\Omega_\gamma\,\rho_{0\mathrm{C}} \times \left(\frac{R_0}{R}\right)^4 . \tag{10.12}$$

According to Refs. [1-6], in the early universe, between photons and monopoles, we have assumed the new thermal equilibrium

$$E_0(\mathrm{M}) = kT_{\mathrm{M}} = (6.849 \pm 0.063)\cdot 10^{26}\ \mathrm{eV} \tag{10.13}$$

with the photon temperature

$$T_{\mathrm{M}} = E_0(\mathrm{M})/k = 7.948\cdot 10^{30}\ \mathrm{K}\,, \tag{10.14}$$

so that Eq. (3.13) of Ref. [6] yields

$$\rho_\gamma(T_{\mathrm{M}})\,c^2 = 4.722\cdot 10^{-3}\left(\frac{T_{\mathrm{M}}}{\mathrm{K}}\right)^4 \mathrm{eV\,cm^{-3}} =$$

$$= 1.884\cdot 10^{121}\ \mathrm{eV\,cm^{-3}}\,. \tag{10.15}$$

Then, using Eqs. (10.8) to (10.12) for photons at $N(T) = 2$, we find

$$t_\gamma = \frac{(2.42035 \pm 0.00015)}{\sqrt{N(T)}}\left(\frac{\mathrm{MeV}}{kT_{\mathrm{M}}}\right)^2 \mathrm{s} = 3.648\cdot 10^{-42}\ \mathrm{s}\,, \tag{10.16}$$

$$\widetilde{R}_\gamma = c\,(4\,\Omega_\gamma)^{1/4}\,(t_{\mathrm{Pl}}\,t_\gamma)^{1/2} = R_{\mathrm{Pl}} = 1.6162\cdot 10^{-33}\ \mathrm{cm}, \tag{10.17}$$

$$R_\gamma = c\,(4\,\Omega_\gamma)^{1/4}\left(\frac{t_\gamma}{H_0}\right)^{1/2} = \widetilde{R}_0 = 4.713\cdot 10^{-3}\ \mathrm{cm}, \tag{10.18}$$

$$\widetilde{\rho}_\gamma(\widetilde{R}_\gamma)\,c^2 = \Omega_\gamma\,\rho_{0\mathrm{C}}\,c^2\left(\frac{\widetilde{R}_0}{\widetilde{R}_\gamma}\right)^4 = 1.883\cdot 10^{121}\ \mathrm{eV\,cm^{-3}} \tag{10.19}$$

and

$$\rho_\gamma(R_\gamma)\,c^2 = \Omega_\gamma\,\rho_{0\mathrm{C}}\,c^2\left(\frac{R_0}{R_\gamma}\right)^4 = 1.883\cdot 10^{121}\ \mathrm{eV\,cm^{-3}}. \tag{10.20}$$

For these photons, because of $t_\gamma > t_{\mathrm{Pl}}$, we assume that $R_\gamma = \widetilde{R}_0$ describes the beginning of the late universe, i.e. therefore the same argument must be valid for $R_\gamma = (1+z_{\mathrm{M}})\,\widetilde{R}_\gamma$ because of the inflation (see above), so that only at the particle energy densities (10.19) the factor $1/(1+z_{\mathrm{M}})^3$ is effective. Thus, the result (10.20) dominates alone for photons, so that we can neglect the photon energy densities (10.19) of the classical universe {because of the factor $1/(1+z_{\mathrm{M}})^3$ } as well as (10.2) and (10.5) of the quantum universe.

Taking Eqs. (10.8) to (10.12) for protons at $N(T) = 205/4$, we get

$$t_{\mathrm{p}} = \frac{(2.42035 \pm 0.00015)}{\sqrt{N(T)}}\left(\frac{\mathrm{MeV}}{E_0(\mathrm{p})}\right)^2 \mathrm{s} = 3.840\cdot 10^{-7}\ \mathrm{s}, \tag{10.21}$$

$$\widetilde{R}_{\mathrm{p}} = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\,(t_{\mathrm{Pl}}\,t_{\mathrm{p}})^{1/2} = 1.180\cdot 10^{-15}\ \mathrm{cm}, \tag{10.22}$$

$$R_{\mathrm{p}} = c\,(2\,N(T)\,\Omega_\gamma)^{1/4}\left(\frac{t_{\mathrm{p}}}{H_0}\right)^{1/2} = 3.441\cdot 10^{15}\ \mathrm{cm}, \tag{10.23}$$

$$\widetilde{\rho}_{\mathrm{p}}(\widetilde{R}_{\mathrm{p}})\,c^2 = \frac{1}{2}N(T)\,\Omega_\gamma\,\rho_{0\mathrm{C}}\,c^2\left(\frac{\widetilde{R}_0}{\widetilde{R}_{\mathrm{p}}}\right)^4 = 1.700\cdot 10^{51}\ \mathrm{eV\,cm^{-3}} \tag{10.24}$$

and

$$\rho_{\mathrm{p}}(R_{\mathrm{p}})\,c^2 = \frac{1}{2}N(T)\,\Omega_\gamma\,\rho_{0\mathrm{C}}\,c^2\left(\frac{R_0}{R_{\mathrm{p}}}\right)^4 = 1.701\cdot 10^{51}\ \mathrm{eV\,cm^{-3}}\,. \quad (10.25)$$

For these protons, because of $t_{\mathrm{p}} >> t_{\mathrm{Pl}}$, we see that $R_{\mathrm{p}} = 3.441\cdot 10^{15}$ cm describes the late universe, i.e. therefore the same argument must be valid for $R_{\mathrm{p}} = (1+z_{\mathrm{M}})\,\tilde{R}_{\mathrm{p}}$ because of the inflation, which however is ineffective, since the protons are formed after the inflation because of $R_0 = (1+z_{\mathrm{M}})\,\tilde{R}_0$.

Consequently, the results (10.24) and (10.25) are valid for protons, so that they dominate, since we can neglect the proton energy density (10.6), resulted from the quantum universe.

Using the redshift condition

$$1+z_{\mathrm{p}} = \frac{E_0(\mathrm{p})/k}{T_0} = 3.995\cdot 10^{12}\,, \quad (10.26)$$

the scale factors (10.22) and (10.23) can be formed also by

$$\tilde{R}_{\mathrm{p}} = \frac{\tilde{R}_0}{1+z_{\mathrm{p}}} = 1.180\cdot 10^{-15}\ \mathrm{cm} \quad (10.27)$$

and

$$R_{\mathrm{p}} = \frac{R_0}{1+z_{\mathrm{p}}} = 3.441\cdot 10^{15}\ \mathrm{cm}\,. \quad (10.28)$$

These results (10.26) to (10.28) justify not only the new inflation model but also the derivation of $\tilde{R}_0$ and R_0 in their meaning for the redshift conditions (see Eqs. (4.14) and (4.15)), i.e. they confirm also the new thermal equilibrium [1-6].

Applying Eqs. (10.8) to (10.12 for monopoles at $N(T) = 1/2\,\Omega_\gamma$, we obtain

$$t_{\mathrm{M}} = \frac{(2.42035 \pm 0.00015)}{\sqrt{N(T)}}\left(\frac{\mathrm{MeV}}{E_0(\mathrm{M})}\right)^2 \mathrm{s} = t_{\mathrm{Pl}} = 5.391\cdot 10^{-44}\ \mathrm{s}\,, \quad (10.29)$$

$$\tilde{R}_{\mathrm{M}} = c\,(t_{\mathrm{Pl}}\,t_{\mathrm{M}})^{1/2} = R_{\mathrm{Pl}} = 1.6162\cdot 10^{-33}\ \mathrm{cm}\,, \quad (10.30)$$

$$R_{\mathrm{M}} = c\left(\frac{t_{\mathrm{M}}}{H_0}\right)^{1/2} = \widetilde{R}_0 = 4.713\cdot 10^{-3}\ \mathrm{cm}, \tag{10.31}$$

$$\widetilde{\rho}_{\mathrm{M}}(\widetilde{R}_{\mathrm{M}})\,c^2 = \frac{1}{4}\rho_{0\mathrm{C}}\,c^2\left(\frac{\widetilde{R}_0}{\widetilde{R}_{\mathrm{M}}}\right)^4 = 8.622\cdot 10^{124}\ \mathrm{eV\,cm^{-3}} \tag{10.32}$$

and

$$\rho_{\mathrm{M}}(R_{\mathrm{M}})\,c^2 = \frac{1}{4}\rho_{0\mathrm{C}}\,c^2\left(\frac{R_0}{R_{\mathrm{M}}}\right)^4 = 8.622\cdot 10^{124}\ \mathrm{eV\,cm^{-3}}. \tag{10.33}$$

For these monopoles, because of $t_{\mathrm{M}} = t_{\mathrm{Pl}}$, we assume the new fact that $R_{\mathrm{M}} = 4.713\cdot 10^{-3}$ cm (see Eq. (10.31)) describes the early universe, i.e. therefore the same argument must be valid for $R_{\mathrm{M}} = (1+z_{\mathrm{M}})\,\widetilde{R}_{\mathrm{M}}$ because of the inflation, so that here at the particle energy densities (10.33) the factor $1/(1+z_{\mathrm{M}})^3$ is effective, whereas the particle energy density (10.32) must decrease via the factor $1/(1+z_{\mathrm{M}})^6$.

Consequently, the result (10.33) yields the particle energy density of the monopoles directly after the inflation, so that we find

$$\rho^*_{\mathrm{M}}(\widetilde{R}_0)\,c^2 = \frac{\rho_{\mathrm{M}}(R_{\mathrm{M}})\,c^2}{(1+z_{\mathrm{M}})^3} = 3.477\cdot 10^{33}\ \mathrm{eV\,cm^{-3}}, \tag{10.34}$$

which dominate the monopole energy density (10.7), arisen from the quantum universe.

In contrast to the result (10.34), the particle energy density (10.32) gives the present monopole energy density of the classical universe to

$$\rho_{\mathrm{M}}(R_0)\,c^2 = \frac{\widetilde{\rho}_{\mathrm{M}}(\widetilde{R}_{\mathrm{M}})\,c^2}{(1+z_{\mathrm{M}})^6} = 1.402\cdot 10^{-58}\ \mathrm{eV\,cm^{-3}}. \tag{10.35}$$

The result (10.34), multiplied by the expansion factor $(\widetilde{R}_0/R_0)^3$, confirms the outcome (10.35) because of

$$\rho_{\mathrm{M}}(R_0)\,c^2 = \rho^*_{\mathrm{M}}(\widetilde{R}_0)\,c^2(\widetilde{R}_0/R_0)^3 = 1.402\cdot 10^{-58}\ \mathrm{eV\,cm^{-3}}. \tag{10.36}$$

Taking the data of Table III and the expansion factor $(R_\gamma/R_0)^3$ together with the kinetic photon and graviton energy (3.6) because of $kT = E_0(\gamma)$ and

$kT = E_0(\mathrm{G})$ as well as the photon energy density (10.20) and the sum $\Omega_{\mathrm{dm}} + \Omega_\Lambda$ of the dark matter and dark energy because of the three sterile neutrinos (see also Eq. (8.9)), we obtain semi-empirically the present photon energy density (8.21) for the classical universe by the plausible expression

$$\rho_\gamma c^2 = \Omega_\gamma \rho_{0\mathrm{C}} c^2 = \rho_0(\gamma) c^2 = \rho_\gamma(R_\gamma) c^2 \times$$

$$\times (\Omega_{\mathrm{dm}} + \Omega_\Lambda) \frac{E_{\mathrm{K}}(\gamma) + E_{\mathrm{K}}(\mathrm{G})}{kT_0} \left(\frac{R_\gamma}{R_0}\right)^3 \cong 0.26\,\mathrm{eV\,cm^{-3}}. \qquad (10.37)$$

Analogously, for the classical universe, the expression (10.19), which must be multiplied by the factor $1/(1+z_{\mathrm{M}})^3$ because of the inflation (see above), yields also the present photon energy density

$$\rho_\gamma c^2 = \Omega_\gamma \rho_{0\mathrm{C}} c^2 = \rho_0(\gamma) c^2 = \frac{\tilde{\rho}_\gamma(\tilde{R}_\gamma) c^2}{(1+z_{\mathrm{M}})^3} \times$$

$$\times (\Omega_{\mathrm{dm}} + \Omega_\Lambda) \frac{E_{\mathrm{K}}(\gamma) + E_{\mathrm{K}}(\mathrm{G})}{kT_0} \cong 0.26\,\mathrm{eV\,cm^{-3}}. \qquad (10.38)$$

Similarly, by the results (10.24) and (10.25), via the expansion factor $(R_{\mathrm{p}}/R_0)^3$, we find plausibly the present baryon energy density of the classical universe (see Table III) to

$$\rho_{\mathrm{b}} c^2 = \Omega_{\mathrm{b}} \rho_{0\mathrm{C}} c^2 = \rho_{\mathrm{p}}(R_{\mathrm{p}}) c^2 \times$$

$$\times \frac{\Omega_{\mathrm{b}}}{1/8\,205\,\Omega_\gamma} \frac{kT_0}{E_0(\mathrm{p})} \left(\frac{R_{\mathrm{p}}}{R_0}\right)^3 \cong 232.7\,\mathrm{eV\,cm^{-3}}. \qquad (10.39)$$

According to Refs. [2-4, 6] a reduced Hubble time can be introduced by

$$t_* = \frac{\tilde{R}_0}{c} = \left(\frac{R_{\mathrm{Pl}}}{c\,H_0}\right)^{1/2} = \left(\frac{t_{\mathrm{Pl}}}{H_0}\right)^{1/2} = \frac{1}{(1+z_{\mathrm{M}})\,H_0} = 1.572 \cdot 10^{-13}\,\mathrm{s}. \qquad (10.40)$$

Using the result (4.27) for $N(T) = 1/2\,\Omega_\gamma$ and $t = t_{\mathrm{Pl}}$ in the classical universe, we find the velocity

$$\dot{\tilde{R}} = 1/2\,c. \qquad (10.41)$$

Via the Hubble parameter

$$H = \frac{\dot{\widetilde{R}}}{\widetilde{R}_0} = \frac{1}{2t_*}, \tag{10.42}$$

we obtain again

$$\widetilde{R}_0 = \dot{\widetilde{R}}\, 2t_* = c\, t_* = 4.713 \cdot 10^{-3}\ \text{cm}. \tag{10.43}$$

Then, taking the results (10.9) to (10.12) for $N(T) = 1/2\,\Omega_\gamma$, we get

$$\widetilde{R}_* = c\,(t_{\text{Pl}}\, t_*)^{1/2} = 2.760 \cdot 10^{-18}\ \text{cm}, \tag{10.44}$$

$$R_* = c\left(\frac{t_*}{H_0}\right)^{1/2} = 8.049 \cdot 10^{12}\ \text{cm}, \tag{10.45}$$

$$\widetilde{\rho}_*(\widetilde{R}_*)\,c^2 = \frac{1}{4}\rho_{0\text{C}}\,c^2\left(\frac{\widetilde{R}_0}{\widetilde{R}_*}\right)^4 =$$

$$= \frac{3c^2}{32\pi G_N t_*^2} = 1.014 \cdot 10^{64}\ \text{eV cm}^{-3} \tag{10.46}$$

and

$$\rho_*(R_*)\,c^2 = \frac{1}{4}\rho_{0\text{C}}\,c^2\left(\frac{R_0}{R_*}\right)^4 =$$

$$= \frac{3c^2}{32\pi G_N t_*^2} = 1.015 \cdot 10^{64}\ \text{eV cm}^{-3}. \tag{10.47}$$

Because of the inflation, the result (10.46) must decrease by the factor $1/(1+z_{\text{M}})^3$, so that we obtain

$$\rho_{**}\,c^2 = \frac{\widetilde{\rho}_*(\widetilde{R}_*)\,c^2}{(1+z_{\text{M}})^3} = 4.092 \cdot 10^{-28}\ \text{eV cm}^{-3}. \tag{10.48}$$

Consequently, because of the particle conservation, in accordance with the expressions (10.35) and (10.36), the result (10.48), multiplied by the factor $kT_0/E_0(\text{M}) = 3.429 \cdot 10^{-31}$ (see also Eq. (10.39)), yields again the present monopole energy density

$$\rho_{\text{M}}(R_0)\,c^2 = \rho_{**}c^2\,\frac{kT_0}{E_0(\text{M})} = 1.403 \cdot 10^{-58}\ \text{eV cm}^{-3}. \tag{10.49}$$

However, using Eq. (10.8) for $N(T)=1/2\,\Omega_\gamma$ and $t=t_*=1.572\cdot10^{-13}\,\mathrm{s}$, we find a new fictitious rest energy

$$E_0(\mathrm{H}^\pm)=E_0(t_*)=kT_*=\left(\frac{1}{2}\left(\frac{90}{8\pi^3}\right)^{1/2}\hbar\,E_{\mathrm{Pl}}\frac{1}{\sqrt{N(T)}\,t_*}\right)^{1/2}=$$

$$=\left(\frac{2.42035\cdot10^{12}\,\mathrm{eV}^2\,\mathrm{s}}{\sqrt{N(T)}}\frac{1}{t_*}\right)^{1/2}\cong4.011\cdot10^{11}\,\mathrm{eV}. \qquad (10.50)$$

We have assumed that the result (10.50) could be responsible for the rest energy $E_0(\mathrm{H}^\pm)\cong4.011\cdot10^{11}\,\mathrm{eV}$ of charged $\mathrm{H}^\pm$ Higgs bosons. Thus, for this interpretation, because of $N(T)=385/4$ of Higgs bosons [2-4, 6, 11], by Eqs. (10.8) or (10.50), we can also expect similarly a new neutral Higgs boson with the rest energy $E_0(\mathrm{H}_1^0)\cong1.253\cdot10^{12}\,\mathrm{eV}$, which is however improbable. However, if the result $E_0(\mathrm{H}_1^0)\cong1.253\cdot10^{12}\,\mathrm{eV}$ is true, we can also assume the coupling $N(T)=(1/2\,\Omega_\gamma\times385/4)^{1/2}=938.8$, so that we can expect analogously the existence of a second new neutral Higgs boson with the rest energy $E_0(\mathrm{H}_2^0)\cong7.089\cdot10^{11}\,\mathrm{eV}$.

This coupling $N(T)=(1/2\,\Omega_\gamma\times385/4)^{1/2}=938.8$ is explained as follows. For the universe, because of the results (10.8) or (10.50), we can write

$$t_*=\frac{2.42035}{\sqrt{1/2\,\Omega_\gamma}}\left(\frac{\mathrm{MeV}}{E_0(\mathrm{H}^\pm)}\right)^2\mathrm{s} \qquad (10.51)$$

and

$$t_*=\frac{2.42035}{\sqrt{385/4}}\left(\frac{\mathrm{MeV}}{E_0(\mathrm{H}_1^0)}\right)^2\mathrm{s}, \qquad (10.52)$$

so that we can form the expression

$$t_*^2=\frac{(2.42035)^2}{(1/2\,\Omega_\gamma\times385/4)^{1/2}}\left(\frac{\mathrm{MeV}}{E_0(\mathrm{H}^\pm)}\right)^2\left(\frac{\mathrm{MeV}}{E_0(\mathrm{H}_1^0)}\right)^2\mathrm{s}^2, \qquad (10.53)$$

which yields

$$E_0(\mathrm{H}_2^0) = \sqrt{E_0(\mathrm{H}^{\pm}) \times E_0(\mathrm{H}_1^0)} =$$

$$= \left(\frac{2.42035}{\sqrt{(1/2\,\Omega_\gamma \times 385/4)^{1/2}}} \frac{\mathrm{MeV}^2}{t_*}\mathrm{s} \right)^{1/2} \cong 7.089 \cdot 10^{11}\,\mathrm{eV}\,. \qquad (10.54)$$

Therefore, alternatively, we can assume that the five predicted observable Higgs boson of the SUSY GUT [23] could be existent as the already observed Higgs boson $E_0(\mathrm{H}^0) \cong 126\,\mathrm{GeV}$ [17] together with the fictitious Higgs bosons $E_0(\mathrm{H}^{\pm}) \cong 401\,\mathrm{GeV}$ and $E_0(\mathrm{H}_1^0) \cong 1253\,\mathrm{GeV}$ as well as $E_0(\mathrm{H}_2^0) \cong 709\,\mathrm{GeV}$.

Taking the Higgs bosons with the rest energy $E_0(\mathrm{H}^0) \cong 1.26 \cdot 10^{11}\,\mathrm{eV}$ and $E_0(\mathrm{H}_1^0) \cong 1.253 \cdot 10^{12}\,\mathrm{eV}$, we can form the two plausible factors

$$(\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/6} \frac{kT_0}{E_0(\mathrm{H}^0)} \cong 1.848 \cdot 10^{-15} \qquad (10.55)$$

and

$$(\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/6} \frac{kT_0}{E_0(\mathrm{H}_1^0)} \cong 1.858 \cdot 10^{-16}\,. \qquad (10.56)$$

Then, the energy density (10.48), multiplied by the two factors (10.55) and (10.56), yields again the present monopole energy density

$$\rho_{\mathrm{M}}(R_0)\,c^2 = (\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/6} \frac{kT_0}{E_0(\mathrm{H}^0)} \times$$

$$\times (\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/6} \frac{kT_0}{E_0(\mathrm{H}_1^0)} \rho_{**} c^2 \cong 1.405 \cdot 10^{-58}\,\mathrm{eV\,cm^{-3}} \qquad (10.57)$$

in accordance with the results (10.35), (10.36) and (10.49), i.e. the outcome (10.57) is again determined by the factor $(\Omega_{\mathrm{dm}} + \Omega_\Lambda)^{1/3}$ (see, e.g., Eq. (8.9)). Consequently, the fictitious result (10.57) supports the upper conclusions for the Higgs bosons.

Introducing this fictitious coupling $N(T) = (1/2\,\Omega_\gamma \times 385/4)^{1/2} = 938.8$ as the statistical value $N(T)$ for the magnetic monopoles with the rest energy $E_0(\mathrm{M}) = 6.849 \cdot 10^{26}\,\mathrm{eV}$, the expressions (10.8) to (10.12) provide

$$t_{\mathrm{M}}^{*} = \frac{2.42035}{\sqrt{(1/2\,\Omega_\gamma \times 385/4)^{1/2}}} \left(\frac{\mathrm{MeV}}{E_0(\mathrm{M})} \right)^2 \mathrm{s} \cong 1.684 \cdot 10^{-43}\,\mathrm{s}, \tag{10.58}$$

$$\widetilde{R}_{\mathrm{M}}^{*} = c\,(2\,(1/2\,\Omega_\gamma \times 385/4)^{1/2}\,\Omega_\gamma)^{1/4}\,(t_{\mathrm{Pl}}\,t_{\mathrm{M}}^{*})^{1/2} =$$

$$= R_{\mathrm{Pl}} \cong 1.6162 \cdot 10^{-33}\,\mathrm{cm}, \tag{10.59}$$

$$R_{\mathrm{M}}^{*} = c\,(2\,(1/2\,\Omega_\gamma \times 385/4)^{1/2}\,\Omega_\gamma)^{1/4} \left(\frac{t_{\mathrm{M}}^{\times}}{H_0} \right)^{1/2} =$$

$$= \widetilde{R}_0 \cong 4.713 \cdot 10^{-3}\,\mathrm{cm}, \tag{10.60}$$

$$\widetilde{\rho}_{\mathrm{M}}^{*}(\widetilde{R}_{\mathrm{M}}^{*})\,c^2 = \frac{1}{2}(1/2\,\Omega_\gamma \times 385/4)^{1/2}\,\Omega_\gamma\,\rho_{0\mathrm{C}}\,c^2 \left(\frac{\widetilde{R}_0}{\widetilde{R}_{\mathrm{M}}^{*}} \right)^4 =$$

$$\cong 8.839 \cdot 10^{123}\,\mathrm{eV\,cm^{-3}} \tag{10.61}$$

and

$$\rho_{\mathrm{M}}^{*}(R_{\mathrm{M}}^{*})\,c^2 = \frac{1}{2}(1/2\,\Omega_\gamma \times 385/4)^{1/2}\,\Omega_\gamma\,\rho_{0\mathrm{C}}\,c^2 \left(\frac{R_0}{R_{\mathrm{M}}^{*}} \right)^4 =$$

$$\cong 8.839 \cdot 10^{123}\,\mathrm{eV\,cm^{-3}}. \tag{10.62}$$

Then, because of the particle conservation, the expressions (10.61) and (10.62) can be multiplied by the factor $1/2(1/2\,\Omega_\gamma \times 385/4)^{1/2}\,\Omega_\gamma = 9.754$, so that we obtain again the results (10.32) and (10.33) via

$$\widetilde{\rho}_{\mathrm{M}}(\widetilde{R}_{\mathrm{M}}) = \frac{\widetilde{\rho}_{\mathrm{M}}^{*}(\widetilde{R}_{\mathrm{M}}^{*})\,c^2}{2(1/2\,\Omega_\gamma \times 385/4)^{1/2}\,\Omega_\gamma} =$$

$$= \frac{1}{4}\rho_{0\mathrm{C}}\,c^2 \left(\frac{\widetilde{R}_0}{\widetilde{R}_{\mathrm{M}}^{*}} \right)^4 \cong 8.622 \cdot 10^{124}\,\mathrm{eV\,cm^{-3}} \tag{10.63}$$

and

$$\rho_{\mathrm{M}}(R_{\mathrm{M}}) = \frac{\rho_{\mathrm{M}}^{*}(R_{\mathrm{M}}^{*})\,c^{2}}{2(1/2\,\Omega_{\gamma}\times 385/4)^{1/2}\,\Omega_{\gamma}} =$$

$$= \frac{1}{4}\rho_{0\mathrm{C}}\,c^{2}\left(\frac{R_0}{R_{\mathrm{M}}^{*}}\right)^{4} \cong 8.622\cdot 10^{124}\ \mathrm{eV\,cm^{-3}}. \qquad (10.64)$$

This particle conservation means the transition from t_{M}^{*} to t_{Pl}, so that the expressions (10.63) and (10.64) permit the same conclusions as at the results (10.32) and (10.33).

These assumptions are reasonable, since at the time $t_{\mathrm{M}}^{*} = 1.684\cdot 10^{-43}\,\mathrm{s}$ (see Eq. (10.58)) we have the formation of the magnetic monopoles with their rest energy $E_0(\mathrm{M}) = 6.849\cdot 10^{26}\,\mathrm{eV}$, whereas for the time $t_{\mathrm{Pl}} = 5.39106\cdot 10^{-44}\,\mathrm{s}$ these magnetic monopoles possess their greatest possible relativistic energy of $E_{\mathrm{Pl}} = \hbar/t_{\mathrm{Pl}} = 1.220932\cdot 10^{28}\,\mathrm{eV}$ (see Eq. (10.29)).

Using the data of Table II and IV, all results, derived in this Sec. 10 for the universe, are also valid for the anti-universe.

11 An exact solution of the monopole problem

Applying the present monopole energy density (10.35), we can determine the present monopole number density of the classical universe to

$$n_{\mathrm{M}} = \frac{\rho_{\mathrm{M}}(R_0)}{E_0(\mathrm{M})} = 2.047\cdot 10^{-85}\ \mathrm{cm^{-3}}. \qquad (11.1)$$

Then, via the known present baryon number density $n_{\mathrm{b}} = 2.482\cdot 10^{-7}\ \mathrm{cm^{-3}}$ (see Ref. [8]), we find the baryon-to-monopole ratio to

$$\eta^{*} = \frac{n_{\mathrm{b}}}{n_{\mathrm{M}}} = 1.213\cdot 10^{78}. \qquad (11.2)$$

Analogously, by the known present photon number density $n_{\gamma} = 410.7\,\mathrm{cm^{-3}}$ (see Ref. [8]), we obtain the photon-to-monopole ratio to

$$\eta^{**} = \frac{n_\gamma}{n_{\mathrm{M}}} = 2.006 \cdot 10^{87} . \tag{11.3}$$

In the classical universe, by the result (6.10), the present total photon number can be defined by

$$N_\gamma = \frac{\frac{1}{2} R_0}{R_{\mathrm{BB}}} = 3.323 \cdot 10^{123} , \tag{11.4}$$

where $R_0 = 1.375 \cdot 10^{28}$ cm is the scale factor of the present universe ($z = 0$).

Analogously, in the classical universe, for $R_{\mathrm{p}} = R_0/(1+z_{\mathrm{p}}) = 3.441 \cdot 10^{15}$ cm (see Eqs. (10.26) and (10.28)) and $\tilde{R}_0 = R_0/(1+z_{\mathrm{M}}) = 4.713 \cdot 10^{-3}$ cm (see Eqs. (4.18) and (4.19)), we find the total proton and monopole number at the time of their formation to

$$N_{\mathrm{p}} = \frac{\frac{1}{2} R_{\mathrm{p}}}{R_{\mathrm{BB}}} = 8.316 \cdot 10^{110} \tag{11.5}$$

and

$$N_{\mathrm{M}} = \frac{\frac{1}{2} \tilde{R}_0}{R_{\mathrm{BB}}} = 1.139 \cdot 10^{93} . \tag{11.6}$$

Then, in the classical universe, for these particles, at the time of their formation, using the results (11.4) to (11.6), we obtain the same value for their total energies (see also Sec. 6 of Ref. [1]) according to

$$E_{\mathrm{tot}} = E_{\mathrm{tot}}(\gamma) = N_\gamma \, kT_0 \cong 7.804 \cdot 10^{119} \text{ eV} , \tag{11.7}$$

$$E_{\mathrm{tot}} = E_{\mathrm{tot}}(\mathrm{p}) = N_{\mathrm{p}} \, E_0(\mathrm{p}) \cong 7.803 \cdot 10^{119} \text{ eV} \tag{11.8}$$

and

$$E_{\mathrm{tot}} = E_{\mathrm{tot}}(\mathrm{M}) = N_{\mathrm{M}} \, E_0(\mathrm{M}) \cong 7.801 \cdot 10^{119} \text{ eV} , \tag{11.9}$$

where kT_0 describes the present thermal energy of the photons. Consequently, we have energy conservation. Thus, similar to the total photon number $N_\gamma = E_{\mathrm{tot}}/kT_0 = E_{\mathrm{H}}/E_0(\gamma)$ [see Eqs. (5.10), (8.7) and (11.7)], we can write

$$N_{\mathrm{p}} = \frac{E_{\mathrm{tot}}}{E_0(\mathrm{p})} = \frac{E_{\mathrm{H}}}{E_0(\gamma)} \frac{kT_0}{E_0(\mathrm{p})} = \frac{N_\gamma}{1+z_{\mathrm{p}}} = \frac{N_\gamma \, kT_0}{E_0(\mathrm{p})} \tag{11.10}$$

and

$$N_{\mathrm{M}} = \frac{E_{\mathrm{tot}}}{E_0(\mathrm{M})} = \frac{E_{\mathrm{H}}}{E_0(\gamma)} \frac{kT_0}{E_0(\mathrm{M})} = \frac{N_\gamma}{1+z_{\mathrm{M}}} = \frac{N_\gamma\, kT_0}{E_0(\mathrm{M})}, \tag{11.11}$$

so that N_{p} and N_{M} exist not only at the time of their formation but also in the present classical universe, i.e. we have also particle conservation.

Therefore, their total present particle number densities can be introduced by

$$n_{\mathrm{M}}^{\mathrm{tot}} = N_{\mathrm{M}}\, n_{\mathrm{M}} = \frac{E_{\mathrm{tot}}}{E_0(\mathrm{M})} n_{\mathrm{M}} = 2.332 \cdot 10^{8}\ \mathrm{cm}^{-3}\ , \tag{11.12}$$

$$n_{\mathrm{b}}^{\mathrm{tot}} = N_{\mathrm{p}}\, n_{\mathrm{b}} = \frac{E_{\mathrm{tot}}}{E_0(\mathrm{p})} n_{\mathrm{b}} = 2.064 \cdot 10^{104}\ \mathrm{cm}^{-3} \tag{11.13}$$

and

$$n_\gamma^{\mathrm{tot}} = N_\gamma\, n_\gamma = \frac{E_{\mathrm{tot}}}{kT_0} n_\gamma = 1.364 \cdot 10^{126}\ \mathrm{cm}^{-3}\ , \tag{11.14}$$

so that we can derive the total particle-to-monopole ratios

$$\eta_{\mathrm{tot}}^{*} = \frac{n_{\mathrm{b}}^{\mathrm{tot}}}{n_{\mathrm{M}}^{\mathrm{tot}}} = \frac{E_0(\mathrm{M})}{E_0(\mathrm{p})} \eta^{*} = \frac{E_0(\mathrm{M})}{E_0(\mathrm{p})} \frac{n_{\mathrm{b}}}{n_{\mathrm{M}}} = 8.851 \cdot 10^{95} \tag{11.15}$$

and

$$\eta_{\mathrm{tot}}^{**} = \frac{n_\gamma^{\mathrm{tot}}}{n_{\mathrm{M}}^{\mathrm{tot}}} = \frac{E_0(\mathrm{M})}{kT_0} \eta^{**} = \frac{E_0(\mathrm{M})}{kT_0} \frac{n_\gamma}{n_{\mathrm{M}}} = 5.849 \cdot 10^{117}\ . \tag{11.16}$$

Consequently, the results (11.15) and (11.16) show precisely and uniquely that presently $8.851 \cdot 10^{95}$ protons or $5.849 \cdot 10^{117}$ photons correspond to one monopole, i.e. the monopoles are no more observable presently. With that, we have solved precisely and uniquely the known monopole problem [7, 9].

Finally, for the total present baryon-to photon ratio, we get

$$\eta_{\mathrm{tot}} = \frac{\eta_{\mathrm{tot}}^{*}}{\eta_{\mathrm{tot}}^{**}} = \frac{n_{\mathrm{b}}^{\mathrm{tot}}}{n_\gamma^{\mathrm{tot}}} = \frac{kT_0}{E_0(\mathrm{p})} \eta = \frac{kT_0}{E_0(\mathrm{p})} \frac{n_{\mathrm{b}}}{n_\gamma} = 1.513 \cdot 10^{-22} \tag{11.17}$$

instead of the known baryon-to photon ratio $\eta = n_{\mathrm{b}}/n_\gamma = 6.05 \cdot 10^{-10}$ [8].

Generally, in the present classical universe, because of the particle (P) rest energies $E_0(\mathrm{P}) \cong 7.804 \cdot 10^{119}\,\mathrm{eV}/N_{\mathrm{P}}$, where N_{P} represents, for example, the

total particle numbers (11.4) to (11.6), we can determine the corresponding particle temperatures to

$$T_{\mathrm{P}} = E_0(\mathrm{P})/k \cong (9.056 \cdot 10^{123}/N_{\mathrm{P}})\,\mathrm{K}\,. \qquad (11.18)$$

Because of the gigantic dimension and the still existent expansion of the present universe, we do not observe here a heat accumulation.

Using the data of Table II to IV, all results, derived in this Sec. 11 for the universe, are also valid for the anti-universe.

12 Summary

For the big bang of the quantum universe (anti-universe), we have confirmed the distance $R_{\mathrm{BB}} = 2.069 \cdot 10^{-98}\,\mathrm{m}$ and the time $t_{\mathrm{BB}} = \pm 6.901 \cdot 10^{-107}\,\mathrm{s}$ of Refs. [1-3], using the rest energy of the photons and gravitons. The vacuum energy density or the cosmological "constant" of the big bang were also confirmed to $\rho_{\mathrm{vac}}(R_{\mathrm{BB}})c^2 = 4.227 \cdot 10^{247}\,\mathrm{eV\,cm^{-3}}$ and $\Lambda = 1.406 \cdot 10^{192}\,\mathrm{m^{-2}}$ (see Refs. [1-3]).

The beginning of the quantum anti-universe (antimatter) and the quantum universe (matter), which are based on the gravitation, is a result of two equivalent energy uncertainties by one quantum fluctuation of the vacuum according to the uncertainty relation in form of $\widetilde{E}_{\mathrm{BB}} = -\hbar/2\,(-t_{\mathrm{BB}})$ and $\widetilde{E}_{\mathrm{BB}} = \hbar/2t_{\mathrm{BB}}$, respectively.

Consequently, the quantum anti-universe has its existence for the negative time direction in the past, whereas the quantum universe exists for the positive time direction in the future, i.e. they expand in opposite time directions.

We have proved that the end of the quantum anti-universe as well as the beginning of the classical anti-universe are simultaneously determined by the negative Planck time in the past, whereas the end of the quantum universe as well as the beginning of the classical universe are simultaneously defined by the positive Planck time in the future.

Therefore, the classical universe and the classical anti-universe, which are based on a new inflation model and the Friedmann-Lemaitre Equations, must expand again in the opposite time directions.

These last results are confirmed by the general description of their radiation-dominated and matter-dominated universe and anti-universe, using correspondingly the known solutions of the universe as starting point, which can be transferred also on the anti-universe.

For the analogous treatment of the final state of the classical anti-universe and universe, we have referred to our early work [1].

With that, we have simply solved the fundamental problem of the separation of antimatter and matter because the unknown anti-universe must be existent in the past, whereas the known universe exists in the future.

We have explained the reason for the existence of the massive rest energy of the photons and the gravitons via a four-dimensional quantum gravity.

We have confirmed that the existence of the universe and the anti-universe can be attributed to zero-point oscillations.

We have shown that at the Planck length the cosmological inflation event at the strong-electroweak symmetry breaking is plausible.

For the total (quantum and classical) universe (anti-universe), via several cosmic energy considerations, which are based on the photons and the gravitons as the 2 decay products of the sterile neutrinos, we have confirmed in detail the proposition of the quantum field theory, which predicts the factor of about 10^{122} according to our considerations via the half Planck energy density and the present vacuum density ρ_Λ.

In the quantum universe (anti-universe), via the quantum gravity [1-3], by which the quantum field theory and the General Relativity are unified, the quantization of the continuous space-time must lead to its separation into its quantized components of the three-dimensional space and the one-dimensional space-dependent time which was attributed to the Planck time.

We have justified the new thermal equilibrium via the massive particle formation in the quantum universe and anti-universe.

Using the new inflation model, we have shown that we can neglect the influence of the particle energy densities of the quantum universe (anti-universe) on the corresponding particle energy densities of the classical

universe (anti-universe), whereat we have estimated a fictitious value of the rest energy of the charged Higgs bosons to $E_0(\mathrm{H}^{\pm}) \cong 4.011 \cdot 10^{11}\,\mathrm{eV}$. On this way, we have also shown that the five predicted observable Higgs boson of the supersymmetric grand unification theory could be existent as the already observed Higgs boson $E_0(\mathrm{H}^0) \cong 126\,\mathrm{GeV}$ together with the fictitious Higgs bosons $E_0(\mathrm{H}^{\pm}) \cong 401\,\mathrm{GeV}$, $E_0(\mathrm{H}_1^0) \cong 1253\,\mathrm{GeV}$ and $E_0(\mathrm{H}_2^0) \cong 709\,\mathrm{GeV}$.

We have solved precisely and uniquely the known monopole problem, since presently $8.851 \cdot 10^{95}$ protons or $5.849 \cdot 10^{117}$ photons correspond to one monopole, so that the monopoles are no more observable presently.

We have proved that the above-described solution of the existence of the universe and the anti-universe explains also the fact that all fundamental physical laws are time-invariant, since they must be valid not only in the universe but also in the anti-universe.

Consequently, all fundamental physical processes of the universe and the anti-universe take place in the positive and the negative time direction, respectively.

In the hitherto existing big bang models [7-13], where the physics of the early universe ($t \leq 3.84 \cdot 10^{-7}\,\mathrm{s}$) is still uncertain, their theories are not well established [7]. These theories predict, e.g., for the elusive dark matter, the existence of many exotic particles [7, 9, 12], for which however there is no experimental evidence [7].

In contrast to these theories, we have shown that our new contributions (see also Refs. [1-6]) complete particularly the picture of the universe and the corresponding anti-universe by the derivation of the parameters of the big bang via the results of the quantum gravity in this work. By these new contributions [1-6], the supersymmetric grand unification theory (SUSY GUT) is now well established at the theories of the early universe.

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ADDITIONAL CONSERVED QUANTITIES OF WEBER'S FORCE LAW?

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Abstract

Weber's force law, derived from Ampère's force law for electrical currents, gives the electrodynamic force between two moving charges. It reduces to Coulomb's law in the static case, but in general it is dependent on the relative velocities and accelerations of the interacting charges. Although it conserves linear and anguler momentum and obeys Newton's 3rd Law, the Runge-Lenz vector is not conserved for it (thus it can explain Mercury's perihelion shift). We investigate whether there are additional symmetries and, by Noether-like theorems, additional conserved quantities of Weber's force law.

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1 Introduction

Ampère's force law [1, 15] expresses the force between electrical current elements $I_1 d\vec{\ell_1}$ and $I_2 d\vec{\ell_2}$ a distance r_{12} apart:

$$I_1 I_2 \frac{\hat{r}_{12}}{r_{12}^2} \left[2(d\vec{\ell_1} \cdot d\vec{\ell_2}) - 3(\hat{r}_{12} \cdot d\vec{\ell_1})(\hat{r}_{12} \cdot d\vec{\ell_2}) \right]. \qquad (1)$$

From this law Weber derived his relative velocity- and acceleration-dependent electrodynamic force between two moving charges [1, 28, 29]; expressed as a system of nonlinear ordinary differential equations (ODEs), it is:

$$\ddot{x}_i = \frac{q_1 q_2 x_i}{r^3} \left[1 - \frac{\zeta}{2c^2} \dot{r}^2 + \frac{\zeta}{c^2} r \ddot{r} \right], \qquad (2)$$

where q_1, q_2 are the interacting charges; $i = 1, 2, 3$; over-dots are time derivatives; x_i the spatial coordinates of the charge; $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$; c the speed of light; and ζ a dimensionless quantity that determines how "Weberian" the force is ($\zeta = 0$ is the Coulomb case); and c is the speed of light *in vacuo*.

Weber's force law is derivable from Neumann's velocity-dependent potential [2, 478]:

$$\frac{q_1 q_2}{r} \left[1 + \frac{\zeta}{2c^2} \dot{r}^2 \right]. \qquad (3)$$

It is repulsive or attractive, depending on the magnitudes of the relative velocities or accelerations; Weber used this fact in his planetary model of the atom [4] to explain—a half-century before Bohr and Rutherford and without any need for a new, nuclear force—how positively charged nucleons can exist in close proximity without being repulsed.

More recently, Frauenfelder and Weber [16] use a Weber Hamiltonian to derive the fine-structure energy levels of a hydrogen atom.

They also show how, according to Carl Neumann, Weber's force can be derived from retarded potentials [14, 148-149; cf. 23, 24].

Weber's force law obeys Newton's 3rd Law.[1] It conserves linear and angular momentum and energy [3, 63-7]. It explains Mercury's perihelion shift [2, 480], so the Runge-Lenz vector is not conserved. But does it have other conservation laws?

2 Symmetry Finding

Sophus Lie originally intended his transformation groups to be a powerful tool for solving differential equations [21, 22], such as by reducing their order. Previously, Seegers [26] used elliptic functions to solve 2, but we investigate whether symmetry methods are more powerful.

A differential equation admits a group of point transformations if it remains invariant under the operation of the transformation given by the group generators [27, 93-94; 5, 46]:

$$X_n = \xi_n(t, q_i)\frac{\partial}{\partial t} + \eta_n^a(t, q_i)\frac{\partial}{\partial q^a}, \tag{4}$$

where $a = 1, 2, 3$ in the case of 2, corresponding to the x, y, z coordinates. Lie showed that $1 \leq n \leq 8$ [22].

The determining equations [20, 202] — partial differential equations (PDEs) that have to be solved to find the symmetry group corresponding to a system of, in this case, second-order differential equations

$$\ddot{q}^a = \omega^a(t, q^i, \dot{q}^i), \tag{5}$$

where $a, i = 1, \ldots, N$, the number of equations; $_{,t} = \frac{\partial}{\partial t}$; and $_{,c} = \frac{\partial}{\partial q^c}$

[1] cf. 8 for how Ampère's force law obeys Newton's 3rd Law

— are [27, 95]:

$$\begin{aligned}\xi\omega^a_{,t}+\eta^b\omega^a_{,b}+\left(\eta^b_{,t}+\dot{q}^c\eta^b_{,c}-\dot{q}^b\xi_{,t}+\dot{q}^b\dot{q}^c\xi_{,c}\right)\frac{\partial\omega^a}{\partial\dot{q}^b}\\ +2\omega^a\left(\xi_{,t}+\dot{q}^b\xi_{,b}\right)+\omega^b\left(\dot{q}^a\xi_{,b}-\eta^a_{,b}\right)\\ +\dot{q}^a\dot{q}^b\dot{q}^c\xi_{,bc}+2\dot{q}^a\dot{q}^c\xi_{,tc}-\dot{q}^c\dot{q}^b\eta^a_{,bc}+\dot{q}^a\xi_{,tt}-2\dot{q}^b\eta^a_{,tb}-\eta^a_{,tt}=0.\end{aligned} \tag{6}$$

Recommended by Hereman [18, 19] as the best symmetry-finding software, we use the Maple package GeM[2] [9–13] to solve for the ξ_n and η^a_n of admitted generators 4 of 2:

```
restart;
read("gem32_12.mpl");

gem_decl_vars(indeps=[t], deps=[x(t),y(t),z(t)],
freeconst=[c,k,zeta]);

r := t -> sqrt(x(t)^2+y(t)^2+z(t)^2);
rp := t -> diff(r(t),t);
rpp := t -> diff(rp(t),t);
fac := t -> k/r(t)^3
(1 - zeta/(2c^2)r(t)^2 + zeta/c^2r(t)rpp(t));

#Coulomb problem:
#zeta := 0;

gem_decl_eqs([
diff(x(t),t$2) = x(t)fac(t),
diff(y(t),t$2) = y(t)fac(t),
diff(z(t),t$2) = z(t)fac(t)],
solve_for=[diff(x(t),t$2),diff(y(t),t$2),diff(z(t),t$2)]);

#This step uses over 6GB of memory:
det_eqs:=gem_symm_det_eqs([t,x(t),y(t),z(t)]);
```

[2] https://math.usask.ca/~shevyakov/gem/

GeM finds 60 determining equations. Solving them with Maple's built-in PDE solver,

```
sym_components:=gem_symm_components();
symm_sol:=pdsolve(det_eqs):
gem_output_symm(symm_sol);
```

GeM outputs these admitted generators:

$$\begin{aligned} X_1 &= \frac{\partial}{\partial t} \\ X_2 &= -x\frac{\partial}{\partial z} + z\frac{\partial}{\partial x} \\ X_3 &= -x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x} \\ X_4 &= -y\frac{\partial}{\partial z} + z\frac{\partial}{\partial y}, \end{aligned} \tag{7}$$

which are time and rotation symmetries [7, 476]; however, when $\zeta \neq 0$, `pdsolve` does not find the dilation symmetry

$$X_5 = \frac{2}{3}\left(x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} + z\frac{\partial}{\partial z}\right) + t\frac{\partial}{\partial t}. \tag{8}$$

This is strange because the determining equations 6 depend on ζ, so it would seem that 8 would depend on ζ, too.

We also tried solving for higher-order (Lie-Bäcklund) symmetries by making the ξ_n and η_n^a depend not only on t, x, y, z but also on $\dot{x}, \dot{y}, \dot{z}$, by changing

```
det_eqs:=gem_symm_det_eqs([t,x(t),y(t),z(t)]);
```

to

```
det_eqs := gem_symm_det_eqs([t, x(t), y(t), z(t),
diff(x(t), t), diff(y(t), t), diff(z(t), t)])
```

After running for about a day, Maple did not find any solutions for additional symmetries.

2.1 Conservation laws

We also tried to find conservation laws [6, 38-120, 363-4] using the following GeM functions:

```
det_eqs:=gem_conslaw_det_eqs([t,x(t),y(t),z(t)]);
CL_multipliers:=gem_conslaw_multipliers();
multipliers_sol:=pdsolve(det_eqs);
gem_get_CL_fluxes(multipliers_sol);
```

But Maple was unable to find any conservation laws, unless we initially set $\zeta = 0$, the Coulomb case.

3 Conclusion

Weber's force law might have additional symmetries and conservation laws, but we were not able to find them. Perhaps a new mathematics, like Lie-Santilli theory [17], is necessary to make finding solutions to the nonlinear ODE 2 more tractable. Santilli [25, 9:18] seems to allude to the possibility that isomathematics can linearize [cf. 20, 214-8] all(?) nonlinear DEs.

Other future possibilities include applying Weber's force law to cold fusion; cf. the lectures by Indrani B. Das Sarma and Simone Beghella Bartoli at this conference.

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ANOMALOUS MAGNETIC MOMENTUM OF MUONS IN FNAL ON THE BASIS OF QUANTIZE SPACE AND TIME THEORY

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Abstract

In this paper by using the quantized space and time theory we explain why magnetic momentum of muons in Fermilab National Accelerator Laboratory (FNAL) has anomaly and is not compatible with the predicted amounts in the standard model. Also, the speed of muons in cyclotron will be calculated.

Key words: Muon, momentum, anomalous, time quanta, space quanta

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1-Introduction:

Recent accurate measurement in FNAL on muons shows a $\sigma = 4.2$ deviation from predicted amount on magnetic momentum of muon[1].In this study we are going to explain this phenomenon on the basis of quantized space and time theory [2]. There are also some studies on Interpretation of this subject. Professor Ruggero Santilli in his recent study has explain that how EPR-entanglement support the anomalous in magnetic momentum in FNAL.[3].

One of the achievements of the space and time theory is "Charge formula", It shows that how charged subatomic particles affect on around the time-space quanta and exchange energy with electromagnetic fields with relativity.

$$q = \frac{\sqrt{8\pi\varepsilon_0 m_0 c^2 l_0}}{\sqrt[4]{1-\frac{v^2}{c^2}}} C = \frac{1\bullet 602\times 10^{-19}}{\sqrt[4]{1-\frac{v^2}{c^2}}} C \qquad (1)[2]$$

$$l_0 = 1\bullet 409\times 10^{-15} m \qquad (2)[2]$$

$$m_0 = 9\bullet 10938188\times 10^{-31} kg \qquad (3)[2]$$

Where m_0 is the initial active mass, and l_0 is length quanta around the particle.

By using this formula it can be found that, time- space around the particles bend by the energy of the charged particles and convert to eight arches by the length of $(\pi \times l_0)$.This formula is regarded to relativity because the length around the particle doesn't decrease and merely changes to arches, but the internal active mass of charged particle increases under the relativity rules. In the standard model, the charge of elementary particle is constant. And this concept makes the anomaly in magnetic momentum results of muon in FNAL.

2. Calculation of the muon frequency in cyclotron

At first we should calculate the speed of muon which created in the electromagnetic field of FNAL. In this experiment the momentum of 3,1 Gev/c apply on each muon. By using two following formulas {(4) and (5)} we can calculate the basic speed of muon in cyclotron.

$$p_{\max} = \frac{h \times \sqrt{1 - \frac{v^2_{\max}}{c^2}}}{n \times l_0 \times \sqrt{1 - \frac{v^2}{c^2}}} \qquad (4)[2]$$

$$E_{\max} = \frac{p_{\max} c^2}{v_{\max}} \qquad (5)[2]$$

Vmax(under the radical): $[0,1] \times v$ (6)[2]

$$r = n \times l_0 \qquad (7)[2]$$

$$r = \sqrt[3]{\frac{M_\mu}{M_e}} \qquad (8)$$

$$r = \sqrt[3]{\frac{1.88353109 \times 10^{-28} kg}{9.10938188 \times 10^{-31} kg}} \approx 6 \times l_0 \qquad (9)$$

V max:The maximum speed of charged particle,
h: plank constant,
P max: the maximum of momentum of each muons .
r: the radius of mouns.
M_e :mass of electron
M_μ mass of muon
The equ 4 and 5, have the main role in calculation of energy exchange and particle momentum in the quantized space and time theory.
Vmax under radical is a constant and quantized number and it appears in the speed of zero and the exact amount of the maximum speed. The following formula is used to calculate the speed of the muons in FNAL.

$$E_{\max} = \frac{p_{\max} c^2}{v_{\max}} \qquad (10)[2]$$

$$p_{\max} = \frac{h}{n \times l_0 \times \sqrt{1 - \frac{v^2}{c^2}}} \qquad (11)$$

The maximum momentum (Pmax) in a determined speed is equal to a momentum which can move the center of muons by the distance of their radius.

Because the momentum 3.1Gev/c is given to the muons in the laboratory, we use the mentioned formula to theoretically obtain the base speed of the muons from the electromagnetic field.

$$p_{\max} = \frac{h}{n \times l_0 \times \sqrt{1 - \frac{v^2}{c^2}}} = 3 \bullet 1 Gev/c \qquad (12)$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{3 \bullet 1 G \times e \times 6 \times l_0}{h \times c} = 21 \bullet 137 \qquad (13)$$

γ = Lorentz Impact factor.

Thus, we have

$$v = 299456761 \bullet 2 m/s \qquad (14)[1]$$

The exact base speed is obtained from the exchange of muon particles momentum with the electromagnetic field of device formula which is impressive.
Using this speed, the period and the base frequency of muons in cyclotron are obtained.

$$T = \frac{7 \bullet 112 \times 2 \times \pi}{299456761 \bullet 2m/s} = 149 \bullet 2ns \qquad (15)[1]$$

$$f = 6701514.Hz \qquad (16)[1]$$

Radius of central orbit of cyclotron=7.112m [1]

Which unbelievably corresponds to the base period level in the FNAL experiment ($T = 149 \bullet 2ns$) [1].

3. Theoretic calculation of muons magnetic momentum anomaly in FNAL

Supposedly, if two muons in a cyclotron stand at a distance of one space quantum, with the momentum exchanged from the first muon to the next one, beings in the motion direction and in the electromagnetic fields direction, this momentum equals,

$$r_{\mu 1} + r_{\mu 2} = 12 \times l_0 \qquad (17)$$

$$E_{\mu-\mu} = \frac{p_{\max} c^2}{v_{\max}} = \frac{h \times c}{n \times l_0 \times \sqrt{1 - \frac{v^2}{c^2}}} = 1 \bullet 4Gev \qquad (18)$$

$$p_{\max} = 3.1Gev/c + 1.4Gev/c \qquad (19)$$

However, the maximum possible energy of a muon is 4.3Gev [4]and the obtained level is greater than that of the maximum muon energy; hence, the speed is said to have reached its maximum level ($v = v_{\max} = 299792407.5m/s$), [4]increasing the radiation of the energy on the base of relativity charge formula.
Thus, the predicted speed is lower than that in the cyclotron at the standard model and this discrepancy of speed equals,

$$\Delta v = 299792407.5 - 299456761.2 = 335646 \bullet 2m/s \qquad (20)$$

Now, assuming the first muons constant and the second ones as receiving the momentum,(The consequenced equations have been presented in a previous study of the author by the title of " solving Einstein twins' paradox "[5].
The anomalous in the magnetic momentum of muons in cyclotron obtain by using charge formula. as follows,

$$q_2 = \frac{1 \bullet 602176462 \times 10^{-19}}{\sqrt[4]{1 - \frac{335646.2^2}{c^2}}} = 1 \bullet 602176964 \times 10^{-19} C \qquad (21)$$

$$\frac{q_2}{q_1} = 1 \bullet 000000313 \qquad (22)$$

By applying the maximum speed in the following formula, the anomalous frequency of muons and their actual period in the FNAL cyclotron will be:

$$T' = \frac{7 \bullet 112 \times 2 \times \pi}{299792407 \bullet 5} = 149 \bullet 05ns$$
$$f' = 6708864.4Hz \qquad (23)$$

Using the $\frac{q_2}{q_1} = 1 \bullet 000000313$ coefficient, the anomalous magnetic momentum of muons in FNAL is estimated

$$\mu_\mu = g(\frac{q}{2m_\mu})\vec{s}$$

Where

$$g = 2(1 + a_\mu) \qquad (24)[1]$$

$$\mu'_\mu = g(\frac{1 \bullet 000000313 \times q_1)\vec{s}}{2m_\mu}) \qquad (25)$$

Using the anomalous magnetic momentum of muons μ'_μ, the angular speed anomaly and detectors anomaly can be determined, yielding the total anomaly in the FNAL experiment.

Conclusion: If the charge of elementary particles is not taken into relativity account, in high-energy experiments, it would be impossible to calculate the energy exchange of sub-atomic particles with the surrounding time and space. The time and space quantization theory is advantageous in that it provides exact energy calculations numbers.

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S. Baeßler,47,c I. Bailey,19,d V. A. Baranov,17 E. Barlas-Yucel,37 T. Barrett,6 E. Barzi,7 A. Basti,11,32 F. Bedeschi,11

A. Behnke,22 M. Berz,20 M. Bhattacharya,43 H. P. Binney,48 R. Bjorkquist,6 P. Bloom,21 J. Bono,7 E. Bottalico,11,32

T. Bowcock,39 D. Boyden,22 G. Cantatore,13,34 R. M. Carey,2 J. Carroll,39 B. C. K. Casey,7 D. Cauz,35,8 S. Ceravolo,9

R. Chakraborty,38 S. P. Chang,18,5 A. Chapelain,6 S. Chappa,7 S. Charity,7 R. Chislett,36 J. Choi,5 Z. Chu,26,e T. E. Chupp,42

M. E. Convery,7 A. Conway,41 G. Corradi,9 S. Corrodi,1 L. Cotrozzi,11,32 J. D. Crnkovic,3,37,43 S. Dabagov,9,f

P. M. De Lurgio,1 P. T. Debevec,37 S. Di Falco,11 P. Di Meo,10 G. Di Sciascio,12 R. Di Stefano,10,30 B. Drendel,7

A. Driutti,35,13,38 V. N. Duginov,17 M. Eads,22 N. Eggert,6 A. Epps,22 J. Esquivel,7 M. Farooq,42 R. Fatemi,38 C. Ferrari,11,14

M. Fertl,48,16 A. Fiedler,22 A. T. Fienberg,48 A. Fioretti,11,14 D. Flay,41 S. B. Foster,2 H. Friedsam,7 E. Frlež,47

N. S. Froemming,48,22 J. Fry,47 C. Fu,26,e C. Gabbanini,11,14 M. D. Galati,11,32 S. Ganguly,37,7 A. Garcia,48 D. E. Gastler,2

J. George,41 L. K. Gibbons,6 A. Gioiosa,29,11 K. L. Giovanetti,15 P. Girotti,11,32 W. Gohn,38 T. Gorringe,38 J. Grange,1,42

S. Grant,36 F. Gray,24 S. Haciomeroglu,5 D. Hahn,7 T. Halewood-Leagas,39 D. Hampai,9 F. Han,38 E. Hazen,2

J. Hempstead,48 S. Henry,44 A. T. Herrod,39,d D. W. Hertzog ,48 G. Hesketh,36 A. Hibbert,39 Z. Hodge,48 J. L. Holzbauer,43

K. W. Hong,47 R. Hong,1,38 M. Iacovacci,10,31 M. Incagli,11 C. Johnstone,7 J. A. Johnstone,7 P. Kammel,48

M. Kargiantoulakis,7 M. Karuza,13,45 J. Kaspar,48 D. Kawall,41 L. Kelton,38 A. Keshavarzi,40 D. Kessler,41

K. S. Khaw,27,26,48,e Z. Khechadoorian,6 N. V. Khomutov,17 B. Kiburg,7 M. Kiburg,7,21 O. Kim,18,5 S. C. Kim,6 Y. I. Kim,5

B. King,39,a N. Kinnaird,2 M. Korostelev,19,d I. Kourbanis,7 E. Kraegeloh,42 V. A. Krylov,17 A. Kuchibhotla,37

N. A. Kuchinskiy,17 K. R. Labe,6 J. LaBounty,48 M. Lancaster,40 M. J. Lee,5 S. Lee,5 S. Leo,37 B. Li,26,1,e D. Li,26,g L. Li,26,e

I. Logashenko,4,b A. Lorente Campos,38 A. Luca`,7 G. Lukicov,36 G. Luo,22 A. Lusiani,11,25 A. L. Lyon,7 B. MacCoy,48

R. Madrak,7 K. Makino,20 F. Marignetti,10,30 S. Mastroianni,10 S. Maxfield,39 M. McEvoy,22 W. Merritt,7

A. A. Mikhailichenko,6,a J. P. Miller,2 S. Miozzi,12 J. P. Morgan,7 W. M. Morse,3 J. Mott,2,7 E. Motuk,36 A. Nath,10,31

D. Newton,39,a,d H. Nguyen,7 M. Oberling,1 R. Osofsky,48 J.-F. Ostiguy,7 S. Park,5 G. Pauletta,35,8 G. M. Piacentino,29,12

R. N. Pilato,11,32 K. T. Pitts,37 B. Plaster,38 D. Počanić,47 N. Pohlman,22 C. C. Polly,7 M. Popovic,7 J. Price,39 B. Quinn,43

N. Raha,11 S. Ramachandran,1 E. Ramberg,7 N. T. Rider,6 J. L. Ritchie,46 B. L. Roberts,2 D. L. Rubin,6 L. Santi,35,8

D. Sathyan,2 H. Schellman,23,h C. Schlesier,37 A. Schreckenberger,46,2,37 Y. K. Semertzidis,5,18 Y. M. Shatunov,4

D. Shemyakin,4,b M. Shenk,22 D. Sim,39 M. W. Smith,48,11 A. Smith,39 A. K. Soha,7 M. Sorbara,12,33 D. Stöckinger,28

J. Stapleton,7 D. Still,7 C. Stoughton,7 D. Stratakis,7 C. Strohman,6 T. Stuttard,36 H. E. Swanson,48 G. Sweetmore,40

D. A. Sweigart,6 M. J. Syphers,22,7 D. A. Tarazona,20 T. Teubner,39 A. E. Tewsley-Booth,42 K. Thomson,39 V. Tishchenko,3

N. H. Tran,2 W. Turner,39 E. Valetov,20,19,27,d D. Vasilkova,36 G. Venanzoni,11 V. P. Volnykh,17 T. Walton,7 M. Warren,36

A. Weisskopf,20 L. Welty-Rieger,7 M. Whitley,39 P. Winter,1 A. Wolski,39,d M. Wormald,39 W. Wu,43 and C. Yoshikawa7

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HOW AND WHY THE DECAY MODES OF BEAUTY QUARKS INTO LEPTONS DECAY IN LHCb ON THE BASIS OF THE QUANTIZED SPACE AND TIME THEORY

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Abstract

In this paper by using the quantized space and time theory, the internal structure of the beauty quark has been analyzed, and the different modes for decay of beauty quark have been predicted theoretically. Also, why the decay of beauty quark into kaon and muons is 88% of its decay into kaon, electron and positron.

Key Words: beauty-quark, HLCb, Length quanta, time quanta, Decay

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1- Introduction

the elementary particles resulted from Decay of the beauty quark in HLCb can be predicted and explained on the basis of the quantized space and time theory. The exact mass of the elementary and the subatomic particles can be calculated theoretically.

The minimum possible length not separable to smaller lengths and it still contains motion is equal to,

l_0 : Length quanta $= 1.409 \times 10^{-15} m$. (1)[1]

The minimum possible time which there is no smaller time interval than it, is equal to,

ΔT_0 : Time quanta $= \frac{l_0}{c} = 0.47 \times 10^{-23}$ (2)[1]

Where $c = 299,792,458^1$ m/s is the speed of light.

2: Analyzing the structure of the beauty quark

2- 1: Analyzing the structure of the beauty quark by using "Higgs boson formula" [2]By using the mass detection of Higgs boson formula, we can determine the mass equivalent energy of beauty quark in an exact range [2]

$$Y = 4E_p + (\pm z)E_{p-p}gluon) - (6 \times (\frac{8\pi\varepsilon_0 m_0 c^2 l_0}{4 \times 2\pi \times \varepsilon_0} \times \frac{1}{L}) \times \frac{1}{4\sqrt{1-\frac{v^2}{c^2}}}) \quad (3)[2]$$

$(\pm 4)E_{p-p}gluon)$: The energy of created gluons between four protons

$4E_p$: The internal energy of four protons and the gluons between each quark.

$(6 \times (\frac{8\pi\varepsilon_0 m_0 c^2 l_0}{4 \times 2\pi \times \varepsilon_0} \times \frac{1}{L}) \times \frac{1}{4\sqrt{1-\frac{v^2}{c^2}}})$: The electromagnetic energy between four protons,

2-2: Determination the minimum and maximum the amounts of beauty quark energy in decay curve.

By using the above mention equation we can assume that three protons have the highest level of internal energy and by collision another proton to them, their speed can be reduced and the beauty quarks is a conclusion of this process. By zeroing the speed in equ 3 we will have:

$$Y = 4E_{0p} + (4 \times E_{p_0 - p_0} gluon) - ((6 \times \frac{8\pi\varepsilon_0 m_0 c^2 l_0}{4 \times 2\pi \times \varepsilon_0} \times \frac{1}{L}) \times \frac{1}{4\sqrt{1 - \frac{0^2}{c^2}}})$$

(4)

$$Y_{\max} = (4 \times M_p \times C^2 \div e) + (16 \times 0.88Gev)$$

$$-(6 \times (0 \bullet 511Mev \frac{l_0}{L})) = 3 \bullet 75Gev + 14 \bullet 08 - (6 \times 0.511Mev) = 17 \bullet 86Gev$$

(5)

$L = l_0$: The distance between protons

$Y_{\max}$: The maximum mass equivalent energy of beauty quarks in the speed of zero

$$Y_{\min} = 17 \bullet 86Gev - (6 \times (0 \bullet 511Mev \times \frac{l_0}{L}) - (4 \times 0 \bullet 88Gev) \qquad (6)$$

$Y\min$: The minimum mass equivalent energy of beauty quarks in speed of zero.

$$l_0 \succ L \succ 2 \bullet 4 \times 10^{-19} m \qquad (7)$$

It should be noticed that the length would be curved .in the baryonic universe the smallest length is the space quanta.

By using the calculated numbers in equ 5 and 6, the minimum and maximum amounts of the beauty quark energy in the Fig: 1, [3] can be obtained.

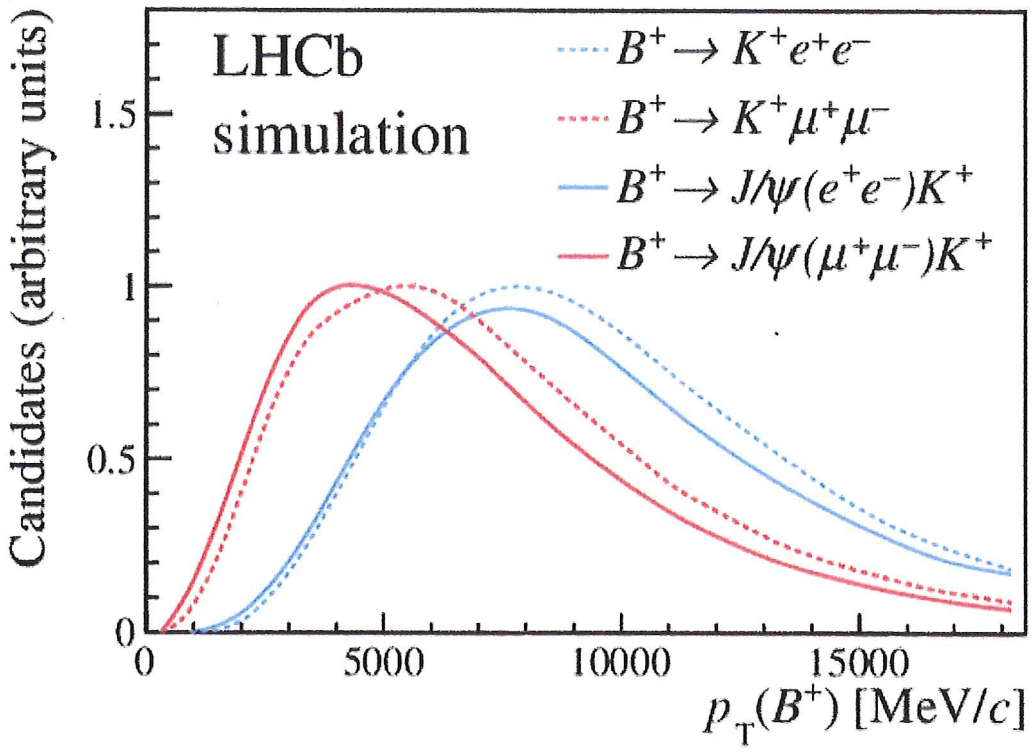

Fig:1 [3]:the structure of beauty quark in at least one time quanta is:

$$\Delta T_0 : \text{Time quanta} = \frac{l_0}{c} = 0.47 \times 10^{-23} \quad (8)$$

The beauty quark can be modeled by four protons. The first three protons make a regular triangular pyramid and the fourth proton can collides to its surfaces or corners of this pyramid.

The major point in the beauty quarks decay is the type of collision of the fourth proton, weather to the surfaces or the corners.

3: Theoretical calculation of the ratio of the two different modes of beauty quark

3-1: Calculation of the effective volumes on the type of forth proton collision

$V1$**:** The total volume of the regular triangle pyramid

$V2$ **:** The effective volume to collision of the fourth proton to the corners.

$V3$: The effective volume to collision of the fourth proton to the surfaces of the regular triangle pyramid.
As the time and space are quantized, so the volume is quantized too and in the same way the inertial mass is quantized. The radius of proton is determined by dividing the mass of proton into electron (mass quanta). [1]

$$r_p = (\sqrt[3]{\frac{M_p}{M_e}}) \times l_0 = 12 l_0 \qquad (9)$$

r_p :. The radius of proton
Therefore the amount of the regular triangle sides is equal to:

$$a = 2 \times 2 \times 12 l_0 = 48 l_0 \qquad (10)$$

And only two protons, not more, can exist on each surface.
As there is a gluon between those two protons so we have:

$$a + 1 l_0 = 49 l_0 \qquad (11)$$

$$V1 = \frac{\sqrt{2}}{12} a^3 = \frac{\sqrt{2}}{12} \times 49^3 l^3{}_0 = 13865 {l_0}^3 \qquad (12)$$

$$V2 = 4 \times (\frac{\sqrt{2}}{12} \times (24 \times l_0)^3) = 6516 l_0^3 \qquad (13)$$

$$V3 = V1 - V2 = 13865 l_0^3 - 6516 l_0^3 = 7349 l_0^3 \qquad (14)$$

By knowing these amounts, the ratio of the decay of proton in the case of collision to the aides and the corners will be achieved.

$$R = \frac{6516 {l_0}^3}{7349 l^3{}_0} = 0 \bullet 886 \qquad (15)$$

3-2: theoretical prediction of the modes of beauty quark decay.
The decay of each subatomic particle takes place by energy exchange between elementary particles [1].

$$p_{\max} = \frac{h \times \sqrt{1 - \frac{{v_{\max}}^2}{c^2}}}{l_0 \times \sqrt{1 - \frac{v^2}{c^2}}} \qquad (16)$$

$$E_{\max} = \frac{p_{\max} c^2}{v_{\max}} \qquad (17)$$

P max: The maximum momentum of each elementary particle related to its speed
v max: Maximum speed of charged elementary particles
E max: The maximum radiated energy from an elementary particle in relation with its speed.

3-2-1: The prediction of the type of elementary and subatomic particles from collision of fourth proton to the corners of the regular triangle pyramid and calculation the mass of them.

The first assume is: the fourth proton collides to the corners. So there are three protons on the base of pyramid and their distance to fourth proton is as a diameter of a single proton.

If four active masses of four protons impacted on each other's by using above mentioned formulas we will have:

$$E_{\max} = \frac{P_{\max} \times C^2 \times 3}{v_{\max}} = \frac{3 \times C^2 \times h \times \sqrt{1 - \frac{v_{\max}^{\ 2}}{c^2}}}{24 \times v_{\max} \times l_0 \sqrt{1 - \frac{v^2}{c^2}}} \qquad (18)$$

Because the distance of fourth proton of the three other protons is equal to 24 times of space quanta it can be assumed that:

$$v = v_{\max} = 299792407 \bullet 5 m/s \qquad (19)$$

Then we have:

$$E_{\max} = 1 \bullet 7 \times 10^{-11} j = M_{\mu} \times C^2 j \qquad (20)$$

M_{μ} : The mass of muon.

The mass of muon incredibly has been calculated.

There are three quarks in a proton, and regarding to proton model [1]

Each proton has three active masses, so the beauty quark will have 12 active masses by considering the fourth proton. The second three- active mass of each proton will cause to produce anti-muon. The last three-active mass will create kaon.

In determination of kaon mass it should be considered that 2/3 of proton masses has converted to muons particles. Now by passing a time quanta, the distance of active masses is four space quanta, so the following decay will be occurred.

$$E_{max} = \frac{P_{max} \times C^2 \times 2}{v_{max}} = \frac{2 \times C^2 \times h \times \sqrt{1 - \frac{v_{max}^2}{c^2}}}{4 \times v_{max} \times l_0 \sqrt{1 - \frac{v_{max}^2}{c^2}}} = 439 Mev \quad (21)$$

$$B^+ \rightarrow K^+ \mu^+ \mu^- \quad (22)$$

In kaon production two active masses covert to two quarks u and s and one gluon instead of coefficient 3 in equ 20, coefficient 2 should be used.

3-2-2: The prediction of the type of elementary and subatomic particles from collision of fourth proton to the surfaces of the regular triangle pyramid and calculation the mass of them

In calculation the mass of resulted particles of beauty quark decay, the main point is the determination of the speed of fourth electron in collision to surfaces of the regular triangle pyramid.

If the subatomic particles reach to the distance of space quanta of each other, because of the minimum possible distance of each other, they cannot have any speed in relation with each other's and the electromagnetic force applies strongly.

So the speed of the fourth proton in relation to the pyramid, when its distance to it, is equal to one space quanta will be zero.

By using the equ 18

$$E1_{max} = \frac{P_{max} \times C^2}{v_{max}} = \frac{C^2 \times h \times \sqrt{1 - \frac{v_{max}^2}{c^2}}}{1 \times v_{max} \times l_0 \sqrt{1 - \frac{0^2}{c^2}}} = 8.18 \times 10^{-14} j = M_e \times C^2 \quad (23)$$

M_e: The mass of electron

The mass of the resulted particle of decay is obtained. This particle is electron. According to the symmetry in the quantized space and time theory [1], positron creates simultaneously to electron. As the same way.

3-2-2 production of kaon will be occured after creation of electron - positron couple and the following decay for beauty quark will take place.

$$B^+ \rightarrow K^+ \mu^+ \mu^- \quad (24)$$

4-: Detection the internal energy of leptons in beauty quarks decay.

The number of leptons and kaons can be calculated in LHCb, by assuming a speed limit for each of them. For example the mass equivalent energy of electron and muon by assuming the speed limit will be $E_e = 0 \bullet 02 Gev$ [2] and $E\mu = 4 \bullet 38 Gev$ [2] respectively.

Prediction of the type of elementary particles resulted from beauty quark decay, according to the type of collision of fourth proton to the regular triangle pyramid is much more facilitated..

5-Conclusion:

Explanation of the recent results regarding to decay of beauty quarks to muons, electrons and positrons in LHCb, can be facilitated and done clearly by using the concepts of the quantized space and time theory which can exactly. Predicts the type of resulted particles of beauty quark decay and calculates the exact mass of them.

Further research:

Design of some other possible experiment in LHCb to achieve the experimental result which is supporting and compatible with theories. Achievements.

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EFFECTIVE POTENTIAL OF MUONIC AND TAUONIC ATOMS OF HELIUM AND HELIUM LIKE ATOMS

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Abstract

The effective potential in this work is the sum of zero, first and second order potential obtained by using Poisson bracket. The effective Potential was developed for a system in which electron, muon and tau orbiting around the nucleus of helium and helium like atoms. Kryukov and Oks (2021) developed effective potential for muon only for low order potential but in this work the effective potential was developed for muon and tau of higher order potential of both helium and helium like atoms with example. In addition, the velocity of tau is greater than velocity of muon and electron in a different path around the nucleus of helium and helium like atoms. Also effective potential is directly proportional to eccentricity and momentum of electron. The effective potential is low at zero radian and increase from 0 to -1 radian and 0 to $+1$ radian.

Keywords: Helium and helium like atom, Effective potential, Muon, Poisson bracket, Muon etc.

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1. Introduction

Physics are interested to study the muonic atoms and molecules because these atoms and molecules are used in different field of physics like muon-catalyzed fusion. The mass of muon is about 200 times the mass of electron and the radius of muon is about 200times less than electron radius. Due the negative charge of muon one can replace orbit electron of an atom with other elementary particles like muon. The life time of muon is about $2.197 \times 10^{-6} sec$ which is larger than the fusion of mesic molecules [1, 2, 3]. The calculation for fine structure is also vary with nuclear mass as $\alpha = 200$ for reduced mass of muon with nucleus. For the isotopes of muon-proton atom $\alpha = 185.8$, for muon-deuterium atom $\alpha = 195.74$ and for muon-tritium atom $\alpha = 199.274$ of one electron system [4]. In similar manner one can develop the different parameters like effective potential, orbiting radius, orbiting velocity around the nucleus of muonic, taunic and other elementary particles.

Muon and electron are Dirac particles and differ from each other only in mass. Similarly, tau and other elementary particle with same charge has also consider as Dirac particles. The history of muonic atoms are begin in 1949 and till now is continuous. The study of muon is consider very recently [5, 6, 7]. For muon-nucleus interaction, the interaction was mainly due to electrostatic charge like electron-nucleus system. Muonic atoms is slow down about 2keV and take about 10^{-9} to 10^{-10} sec when hits the target. the transition of all muonic states are unoccupied and the phenomena take place with 10^{-13}sec [8, 9, 10].

The numerical value of eccentricity (e) determine the shape of electron orbit. If we apply Kepler planetary to atomic model, the for elliptical orbit $e < 1$, for circular orbit $e = 1$ and for hyperbola $e > 1$. Kryukov and Oks (2021) develop and model to study the effective potential and frequency ratio (precision frequency to the Kepler frequency), for muonic-helium and helium like atoms. Since helium orbit has 2 electron and if we replace one by muon then both is orbiting around the nucleus due to coulomb potential representing in equation (1) at different radius (r) with $e << 1$,

$$V = -\frac{1}{4\pi\varepsilon_0}\frac{Zq^2}{r} = -\frac{k}{r} \tag{1}$$

On assuming $\frac{1}{4\pi\varepsilon_0}$ unity and $k = Zq^2$ with charge q is charge of electron. The muonic-helium system is assumed as Kepler planetary model, therefore from Kepler model equation of motion is,

$$\frac{l}{r} = 1 + e\cos\theta \tag{2}$$

Here $l = \frac{L^2}{m_r k}, e = \sqrt{1 + \frac{2EL^2}{m_r k^2}}$ is eccentricity in polar coordinate system, E is energy of system, L is angular momentum and m_r is reduced mass of subsystem. Now equation (2) can expressed for $(e \ll 1)$ also as

$$r = r_0(1 + ecos\theta) \tag{3}$$

Here r_0 is radius of circular orbit for $e = 0$ and for planetary system e <<1 is consider nearly circular orbit example for Mercury eccentricity is about e ≈ 0.2 and for Mars is about e ≈ 0.09 [11].

2. Theoretical Framework for Effective Potential

The effective potential for Rydberg electron without time dependent is expressed by Kryukov and Oks as

$$V_{eff}^0 = -\frac{(Z-1)e^2}{r} - \frac{(R_{\mu 0}^2 - ZR_{n0}^2)e^2}{4r^3} \times \quad (3\cos^2\phi - 1) \tag{4}$$

In general, $R_\mu = R_{\mu 0}(1 + \epsilon cos\Omega t), \quad R_n = R_{n0}(1 + \epsilon cos\Omega t), \Omega = \sqrt{\frac{Ze^2}{m_{\mu r}(R_\mu + R_n)^3}} \approx \sqrt{\frac{Ze^2}{m_{\mu r}R_\mu^3}}$, Ω is frequency of revolution of muon aaround the nucleus with center of mass and $m_{\mu r} = \frac{m_\mu m_n}{m_\mu + m_n}$ is reduced mass of pair nucleus–muon [12]. Putting the value of $R_{\mu 0}$ and R_{n0} in (4) the time dependent potential (TDP) obtain as,

$$V(t) = -\frac{(Z-1)q^2}{r} - \frac{\left(\frac{R_\mu^2}{(1+ecos\Omega t)^2} - Z\frac{R_n^2}{(1+ecos\Omega t)^2}\right)q^2}{4r^3}(3\cos^2\phi - 1) \tag{5}$$

On solving equation (5) the TDP is obtain as,

$$V(t) = V_{eff}^0 + \mathrm{W(r, cos\phi)}cos\Omega t \tag{6}$$

Where,

$$V_{eff}^0 = -\frac{(Z-1)q^2}{r} - \frac{(R_\mu^2 - ZR_n^2)q^2}{4r^3}(3\cos^2\phi - 1),$$

and

$$\mathrm{W(r, cos\phi)} = \frac{(R_\mu^2 - ZR_n^2)e^2}{2r^3}(3\cos^2\phi - 1)e.$$

Also the second term of time dependent perturbation is proceed for effective potential because first term is unperturbed and time independent. The unperturbed total Hamiltonian of a system is obtain as

$$H_0 = \frac{p^2}{2m_{er}} - \frac{(Z-1)e^2}{r} \tag{7}$$

The first term represent as kinetic with reduce mass of electron revolving around nucleus-muon pair as $m_{er}^{\mu} = \frac{m_e(m_\mu+m_n)}{m_e+m_\mu+m_n}$ and second term represent as potential energy term. Using last term of equation (6) as $W = \mathrm{W(r,cos\phi)} = \frac{R_\mu^2 e^2}{2r^3}(3\cos^2\phi - 1)q$ and on neglecting the radius of nucleus we get zero order effective potential is obtain as,

$$V_0 = \frac{1}{4\Omega^2}[W,[W,H_0]] = \frac{9R_\mu^4 q^4 \epsilon^2 (9\cos^4\phi - 6cos^2\phi + 1)}{16 m_{er}^{\mu}\Omega^2 r^8} \tag{8}$$

Equation (8) is obtained using Poisson bracket. Similarly, the first and second order effective potential can be obtain as shown in equation (9) and (10), respectively,

$$V_1 = \frac{1}{4\Omega^2}[[W,H_0],[[W,H_0],H_0]]$$

$$= \frac{-1}{4\Omega^2}\left(\frac{3R_\mu^2 q^2 e(\mathbf{3\cos^2\phi - 1})}{2m_{er}^{\mu}}\right)^2 \times$$

$$\left(\frac{12p^2}{m_{er}^{\mu} r^{10}} + \frac{6(Z-1)q^2}{r^{11}}\right) \tag{9}$$

And

$$V_2 = \frac{1}{4\Omega^2}\Big[[W,H_0],\Big[[W,H_0],[[W,H_0],H_0]\Big]\Big]$$

$$= \frac{1}{4\Omega^2}\left(\frac{3R_\mu^2 q^2 e}{2m_{er}^{\mu}}(3\cos^2\phi - 1)\right)^3 \times$$

$$\left[\frac{24p^2}{m_{er}^{\mu} r^{15}} + \frac{66(Z-1)q^2}{r^{17}}\right] \tag{10}$$

The effective potential of system is expressed as

$$V_{eff} = V_{eff}^0 + V_0 + V_1 + V_2$$

$$V_{eff} = -\frac{(Z-1)q^2}{r} - \frac{(R_\mu^2 - ZR_n^2)q^2}{4r^3}(3\cos^2\phi - 1) + \frac{9R_\mu^4 q^4 e^2(3cos^2\phi - 1)}{16m_{er}^\mu \Omega^2 r^8} - \frac{1}{4\Omega^2}\left(\frac{3R_\mu^2 q^2 e(3\cos^2\phi - 1)}{2m_{er}^\mu}\right)^2 \times \left(\frac{12p^2}{m_{er}^\mu r^{10}} + \frac{6(Z-1)q^2}{r^{11}}\right) + \frac{1}{4\Omega^2}\left(\frac{3R_\mu^2 q^2 e}{2m_{er}^\mu}(3\cos^2\phi - 1)\right)^3 \times \left[\frac{24p^2}{m_{er}^\mu r^{15}} + \frac{66(Z-1)q^2}{r^{17}}\right] \quad (11)$$

Using $\Omega^2 \approx \frac{Zq^2}{m_{\mu r}R_{\mu 0}^3}$ and neglecting nucleus radius of equation (11) we get,

$$V_{eff} = -\frac{(Z-1)q^2}{r} - \frac{R_\mu^2 q^2}{4r^3}(3\cos^2\phi - 1) + \frac{9m_{\mu r}R_\mu^7 q^2 e^2(3cos^2\phi - 1)}{16Zm_{er}r^8} - \frac{9m_{\mu r}R_\mu^7 q^2 e^2(3\cos^2\phi - 1)^2}{16Z(m_{er}^\mu)^2} \times \left(\frac{12p^2}{m_{er}^\mu r^{10}} + \frac{6(Z-1)q^2}{r^{11}}\right) + \frac{27m_{\mu r}R_\mu^9 q^4 e^3}{32Z(m_{er}^\mu)^3}(3\cos^2\phi - 1)^3 \left[\frac{24p^2}{m_{er}^\mu r^{15}} + \frac{66(Z-1)q^2}{r^{17}}\right] \quad (12)$$

Using Atomic unit $\hbar = m_q = q = 1$ the equation (12) can be written as,

$$V_{eff}^\mu = -\frac{(Z-1)}{r} - \frac{R_\mu^2}{4r^3}(3\cos^2\phi - 1) + \frac{9m_{\mu r}R_\mu^7 e^2(3cos^2\phi - 1)}{16Zm_{er}^\mu r^8} - \frac{9m_{\mu r}R_\mu^7 e^2(3\cos^2\phi - 1)^2}{16Z(m_{er}^\mu)^2} \times \left(\frac{12p^2}{m_{er}^\mu r^{10}} + \frac{6(Z-1)}{r^{11}}\right) + \frac{27m_{\mu r}R_\mu^9 e^3}{32Z(m_{er}^\mu)^3} \times (3\cos^2\phi - 1)^3 \left[\frac{24p^2}{m_{er}^\mu r^{15}} + \frac{66(Z-1)}{r^{17}}\right] \quad (13)$$

This equation represent the effective potential of muon–nucleon- electron system. Since, the mass of muon is greater than mass of electron. Therefore the muon is more closure to the nucleus than electron and have different orbiting path around the nucleus. So, the speed of the muon is faster than the speed of the electron around the nucleus.

Similarly, equation (14) is obtained for other elementary charge particle tau whose mass is $3477.50m_e$,

$$V_{eff}^\tau = -\frac{(Z-1)}{r} - \frac{R_\tau^2}{4r^3}(3\cos^2\phi - 1) + \frac{9m_{\tau r}R_\tau^7 e^2(3cos^2\phi - 1)}{16Zm_{er}^\tau r^8} - \frac{9m_{\tau r}R_\tau^7 e^2(3\cos^2\phi - 1)^2}{16Z(m_{er}^\tau)^2} \times \left(\frac{12p^2}{m_{er}^\tau r^{10}} + \frac{6(Z-1)q^2}{r^{11}}\right) + \frac{27m_{\tau r}R_\tau^9 e^3}{32Z(m_{er}^\tau)^3} \times (3\cos^2\phi - 1)^3 \left[\frac{24p^2}{m_{er}^\tau r^{15}} + \frac{66(Z-1)q^2}{r^{17}}\right] \quad (14)$$

Here, $m_{\tau r} = \frac{m_\tau m_n}{m_\tau + m_n}, m_{er}^{\tau} = \frac{m_e(m_\tau + m_n)}{m_e + m_\tau + m_n}$, since the orbit is near circular. Therefore one can used an relation, electron revolving with certain frequency around the nucleus as,

$$\omega_e = \frac{n\hbar}{m_e r_e^2} \tag{15}$$

Similarly for muon as,

$$\omega_\mu = \frac{n\hbar}{m_\mu R_\mu^2} \tag{16}$$

To compare the muon and electron frequency, taking the ratio of (15) and (16). Solving for $m_\mu = 200 m_e$,

$$\frac{\omega_e}{\omega_\mu} = \frac{m_\mu R_\mu^2}{m_e r_e^2} = \frac{200 R_\mu^2}{r_e^2} \tag{17}$$

As muon mass is greater than electron. Therefore electron revolved around the nucleus is slower than muon. So, the frequency of muon is greater than electron. Hence, from equation (17),

$$r_e \approx 14 R_\mu \tag{18}$$

This equation represent that radius of electron orbit is greater than radius of muon orbit. Also shows, the radius of electron revolving around the nucleus is 14 times greater than radius of muon, revolving around the nucleus. Using the radius of electron through which electron revolved around nucleus as $r_e = 53pm$. From equation (18), the radius of muon revolving around the nucleus is $3.79pm$. Similarly with the help of (17) and (18), for tau $m_\tau = 3477.50 m_e$,

$$r_e \approx 59 R_\tau \tag{19}$$

This equation represent that radius of electron orbit is greater than radius of tau orbit. Also shows, the radius of electron revolving around the nucleus is 59 times greater than radius of tau, revolving around the nucleus. Using the radius of electron through which electron revolved around nucleus as $r_e = 5.3 \times 10^{-11} m$ from equation (19) the radius of tau revolving around the nucleus is $0.0989 \times 10^{-11} m$. Now from equation (18) and (19),

$$R_\mu = 4.21 R_\tau \tag{20}$$

This equation (20) shows, the radius of muon particle revolving around the nucleus is greater than that tau particles revolving around the nucleus.

Therefore the radius of the same charge particle revolving around the nucleus has an order of radius $r > R_\mu > R_\tau$.

3. Result and discussion

Table 1 and Table 2 represent the isotopic mass of helium and helium like atoms with reduced mass of muon and tau. The reduced mass of muon for lithium ion and helium orbit is ranges from $102.13 MeVc^{-2}$ to $105.08\ MeVc^{-2}$, tau ranges from $1086.24\ MeVc^{-2}$ to$1549\ MeVc^{-2}$. The reduced mass of electron orbiting Tau-Nucleus-Electron and Muon-Nucleus-Electron are about $0.511\ MeVc^{-2}$. More detail of reduction masses for each isotopes of helium atom and lithium ions are represented in Table 1 and Table 2. The tables proof that the develop equation for muon and tau effective potential are better working for helium and helium like atoms for elementary particles of same charge (electron). For this study authors consider mass of electron is $0.511 MeVc^{-2}$, proton is $938.272 MeVc^{-2}$, neutron is $939.565 MeVc^{-2}$, muon is $106 MeVc^{-2}$ and tau is $1777 MeVc^{-2}$.

Table 1: Isotopes of Lithium ion with reduce mass of muon, tau and electron revolving around the nucleus

Lithium ion Isot-opes	Mass ($MeVc^{-2}$)	Reduced mass of muon with nucleus ($MeVc^{-2}$)	Reduced mass of electron with nucleus –muon pair ($MeVc^{-2}$)	Reduced mass of tau with nucleus ($MeVc^{-2}$)	Reduced mass of electron with nucleus –tau pair ($MeVc^{-2}$)
$^{3}Li^{+}$	2794.38	102.13	0.51090998593	1086.24	0.51094288555
$^{4}Li^{+}$	3725.84	103.07	0.51093186401	1203.16	0.51095255236
$^{5}Li^{+}$	4657.30	103.64	0.51094518653	1286.24	0.51095942056
$^{6}Li^{+}$	5588.76	104.03	0.51095415126	1348.30	0.51096455180
$^{7}Li^{+}$	6520.22	104.30	0.51096059580	1396.42	0.51096853103
$^{8}Li^{+}$	7451.68	104.51	0.51096545191	1434.84	0.51097170705
$^{9}Li^{+}$	8383.14	104.68	0.51096924243	1466.20	0.51097430076
$^{10}Li^{+}$	9314.60	104.81	0.51097228341	1492.30	0.51097645885
$^{11}Li^{+}$	10246.06	104.91	0.51097477718	1514.36	0.51097828257
$^{12}Li^{+}$	11177.52	105.00	0.51097685924	1533.25	0.51097984404
$^{13}Li^{+}$	12108.98	105.08	0.51097862378	1549.60	0.51098119604

Table 2: Isotopes of helium with reduce mass of muon, tau and electron revolving around the nucleus

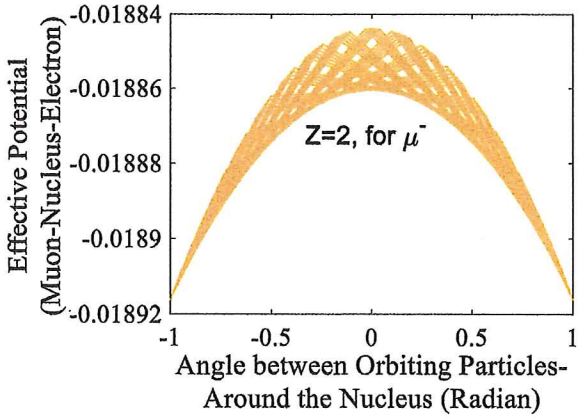
Helium Isotopes	Approximate Isotopes Mass ($MeVc^{-2}$)	Reduced mass muon with nucleus ($MeVc^{-2}$)	Reduced mass of electron with nucleus –muon pair ($MeVc^{-2}$)	Reduced mass tau with nucleus ($MeVc^{-2}$)	Reduced mass of electron with nucleus – tau pair ($MeVc^{-2}$)
^{3}He	2,794.38	102.13	0.51090998593	1086.24	0.51094288555
^{4}He	3,725.84	103.07	0.51093186401	1203.16	0.51095255236
^{5}He	4,657.3	103.64	0.51094518653	1286.24	0.51095942056
^{6}He	5,588.76	104.03	0.51095415126	1348.30	0.5109645518
^{7}He	6,520.22	104.30	0.5109605958	1396.42	0.51096853103
^{8}He	7,451.68	104.51	0.51096545191	1434.84	0.51097170705
^{9}He	8,383.14	104.68	0.51096924243	1466.20	0.51097430076
^{10}He	9,314.6	104.81	0.51097228341	1492.30	0.51097645885

3.1. Speed of the muon, tau and electron around the nucleus of helium and lithium ion isotopes

The study, observation and developed model shows, the velocity of tau > velocity of muon > velocity of electron, for the orbit e<<1. The mass of corresponding elementary particles are as, mass of tau > mass of muon > mass of electron. In addition, the orbital radius for muon, tau and electron was found different as, orbital radius of tau < orbital radius of muon < orbital radius of electron, revolving around the nucleus of helium and lithium ion.

3.2. Effective Potential with orbiting angle of helium and helium like atoms (lithium ion)

Fig.1, show the representation and proof of the developed equation of effective potential for helium and lithium ion (helium like atoms). The effective potential in this work is consider as the sum of zero order, first order and second order potential, calculated by using Poisson bracket. The nature of effective potential is same for helium and lithium ion. Moreover, this model is also work for helium like atoms. The visualization is based with eccentricity($e = 0.1$), reduced mass of muon $103.07 MeVc^{-2}$, reduced mass of tau $1777 MeVc^{-2}$, reduce mass of electron $0.511 MeVc^{-2}$, momentum 2a.u. radius of electron 53pm, radius of tau 0.989pm and radius of muon 3.79pm.



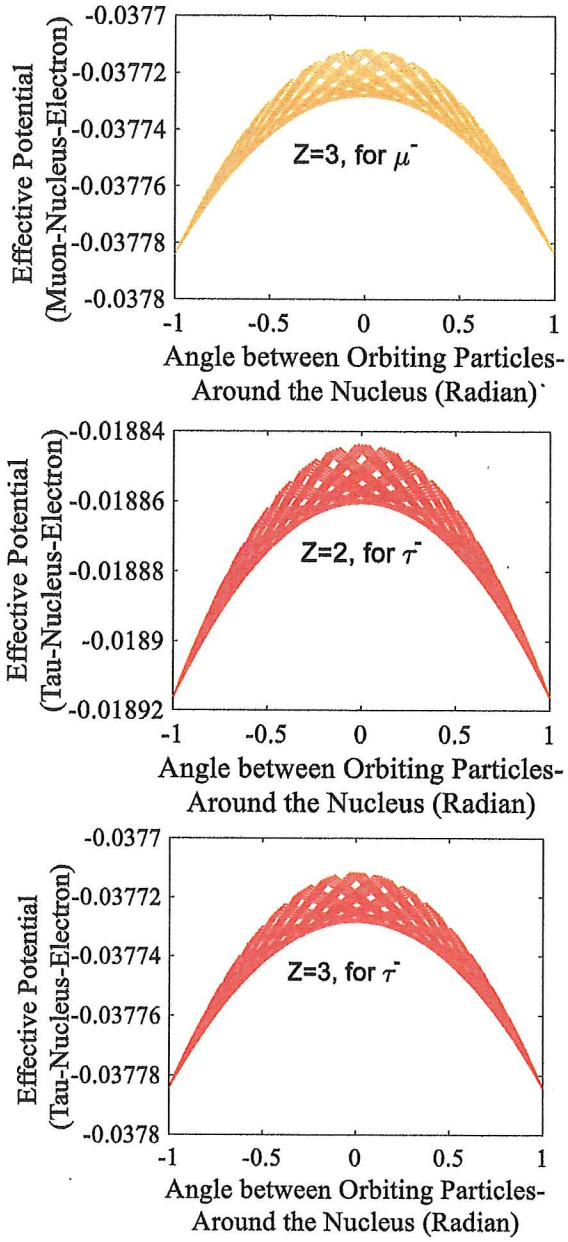
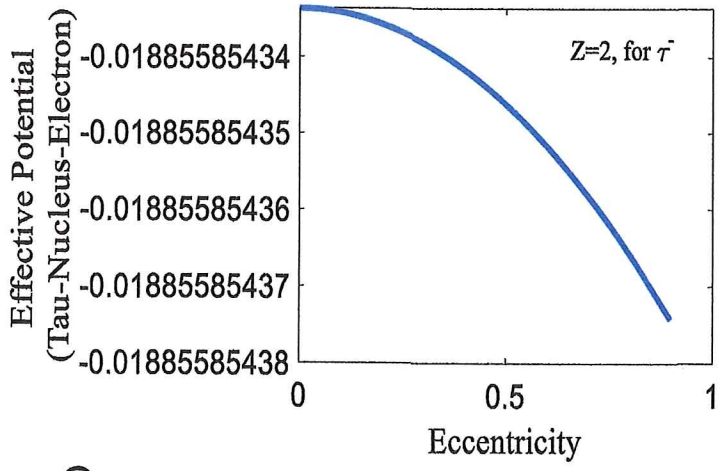


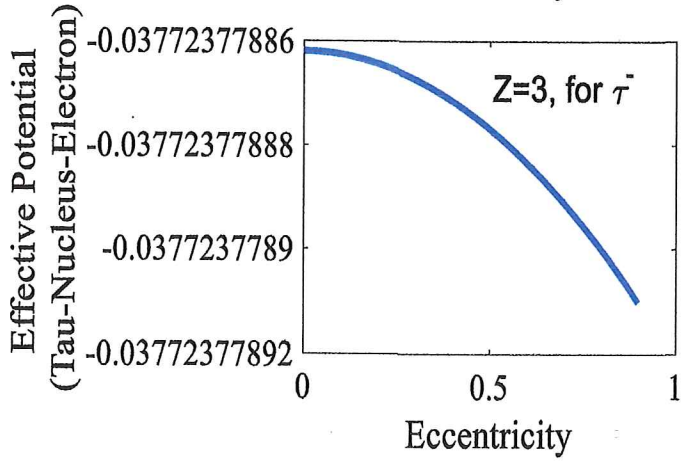
Fig.1: Representation of effective potential with orbiting angle around the nucleus

The effective potential at zero radian is minimum but goes increase as angle decrease 0 to -1 radian and 0 to 1 radian. This is because at zero radian the distance between nucleus and particles (tau, electron, and muon) are apart due to elliptical shape of orbit.

3.3. Effective Potential with eccentricity for helium and helium like atoms (lithium ion)

Fig.2, represent the proof and nature of effective potential for helium and lithium ion within range of 0 to 0.9 because in our case e<<1 (nearly circular orbit). The effective potential goes increase with increasing the eccentricity and the effective potential with eccentricity is very closure as shown in Fig.2. This is because eccentricity value depend upon electron orbit (shape) but not muon and tau, the mass of tau and muon increase the effective potential of the system. The effective potential of tau-nucleus-electron is greater than muon-nucleus-electron for lithium ion. Similarly the effective potential of tau-nucleus-electron is also greater than muon-nucleus-electron for helium atom. The representation is based on 0.35radian (20degree), reduced mass of muon $103.07 MeVc^{-2}$, reduced mass of tau $1777 MeVc^{-2}$, reduce mass of electron $0.511 MeVc^{-2}$, momentum 2a.u. radius of electron 53pm, radius of tau 0.989pm and radius of muon 3.79pm.





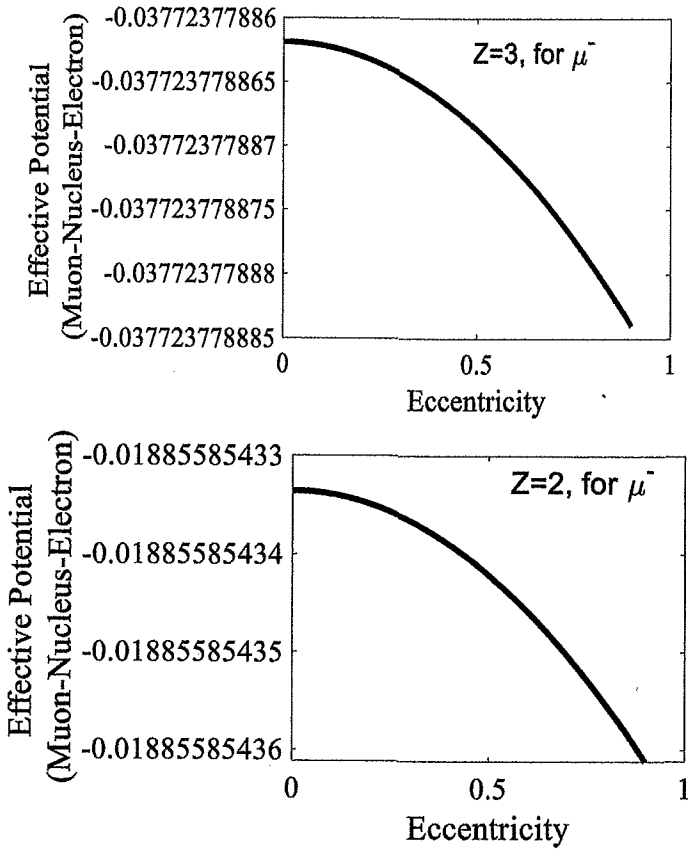
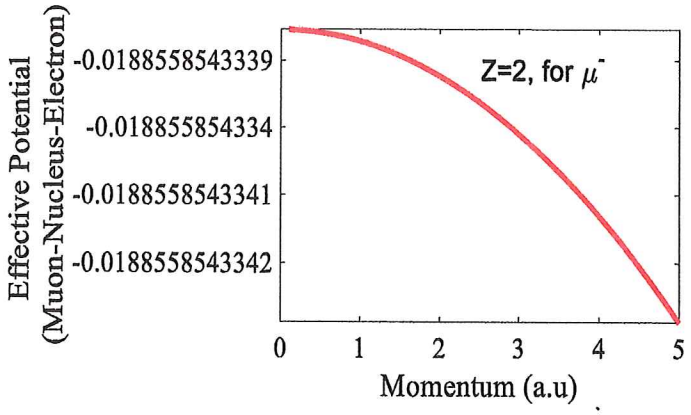
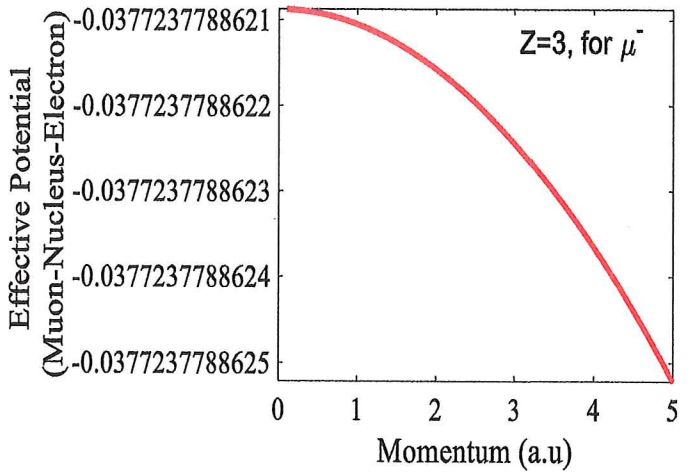


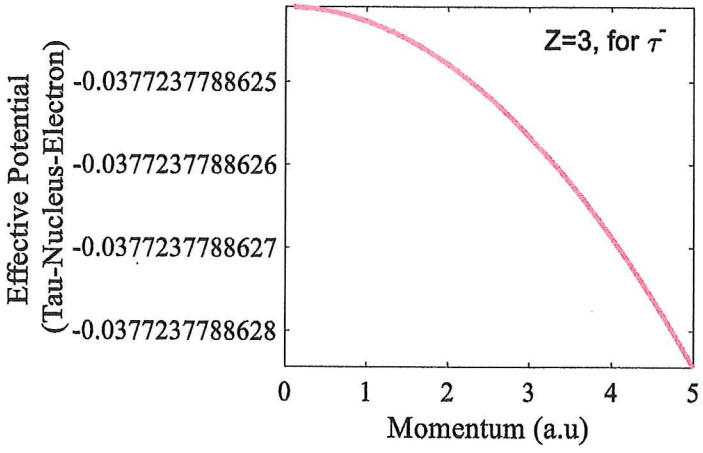
Fig. 2: Representation of effective potential with eccentricity of Tau-Nucleus and Muon-Nucleus of Helium and helium like atoms.

3.4. Effective Potential with momentum of particle on orbit for helium and helium like atoms (lithium ion)

Fig.3, represent the effective potential with momentum of electron in revolving orbit. The nature of potential for tau-nucleus-electron and muon-nucleus-electron for helium and helium like atoms are same. The develop theory above verified that the system develop for effective potential of helium is applicable for lithium ion and helium like atoms, with the elementary particle in the orbit of atom having same charge as electron (like tau and muon). The representation is based on 0.35radian (20degree), reduced mass of muon $103.07 MeVc^{-2}$, reduced mass of tau $1777 MeVc^{-2}$, reduce mass of electron $0.511 MeVc^{-2}$, radius of electron 53pm, radius of tau 0.989pm and radius of muon 3.79pm.


Effective Potential
(Muon-Nucleus-Electron)
-0.0188558543339
-0.018855854334
-0.0188558543341
-0.0188558543342
Z=2, for μ^-
0
1
2
3
4
5
Momentum (a.u)


Effective Potential
(Muon-Nucleus-Electron)
-0.0377237788621
-0.0377237788622
-0.0377237788623
-0.0377237788624
-0.0377237788625
Z=3, for μ^-
0
1
2
3
4
5
Momentum (a.u)


Effective Potential
(Tau-Nucleus-Electron)
-0.0377237788625
-0.0377237788626
-0.0377237788627
-0.0377237788628
Z=3, for τ^-
0
1
2
3
4
5
Momentum (a.u)

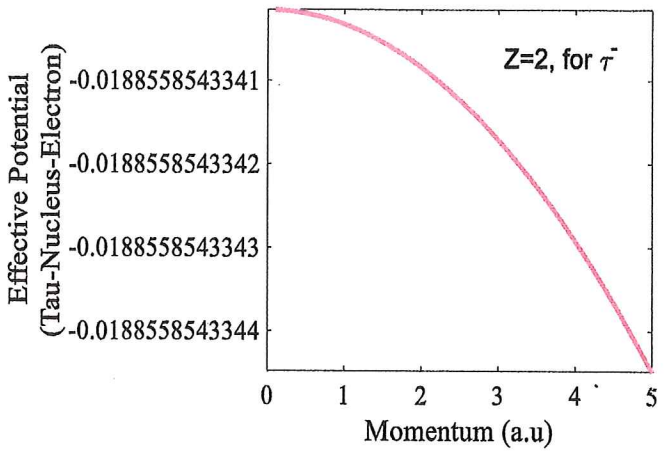


Fig.3: Representation of effective potential with momentum of electron orbiting around Tau-Nucleus and Muon-Nucleus of Helium and helium like atoms.

4. Conclusion

Finally, the developed equation (13) and (14) is the effective potential using Poisson bracket was visualized in Fig.1, Fig.2 and Fig.3. Equation (18) and (19) shows the radius of tau and muon around the nucleus of helium and helium like atoms for $e << 1$ is less than radius of electron around the same nucleus. In addition the velocity of tau is greater than velocity of muon and the velocity of muon is greater than velocity of electron revolving around the same nucleus of helium and helium like atoms. The effective potential increase with increasing the eccentricity and momentum of electron for helium like atoms while the effective potential is minimum at 0 radian and goes increase from 0 to $+1$ and 0 to -1 radian. The developed equation and visualization shows that for particle with same charge of electron and different in masses around the nucleus has different orbit, velocity and potential.

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